

2011 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS

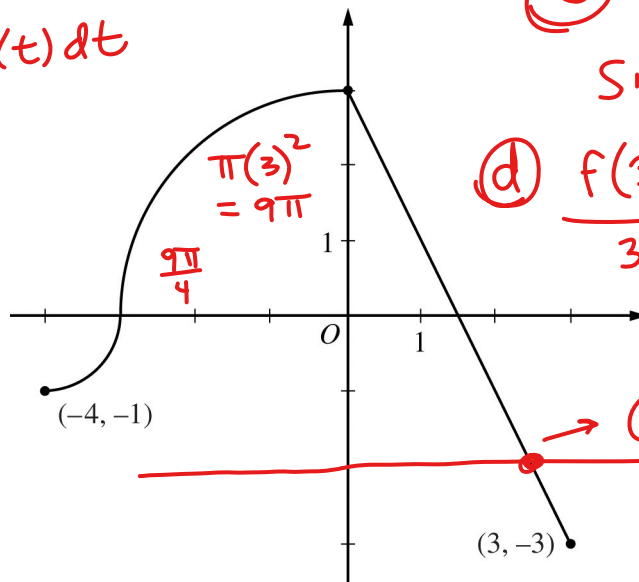
$$g(-3) = 2(-3) + \int_0^{-3} f(t) dt$$

$$= -6 - \frac{9\pi}{4}$$

$$g'(-3) = 2 + f(-3)$$

$$= 2 + 0$$

$$= 2$$



Graph of  $f$

(c)  $g'$  changes sign at  $x=0$

$$(d) \frac{f(3) - f(-4)}{3 - (-4)} = \frac{-3 - (-1)}{3 - (-4)}$$

$$= \frac{-2}{7}$$

$f$  is not differentiable at  $x=0$

4. The continuous function  $f$  is defined on the interval  $-4 \leq x \leq 3$ . The graph of  $f$  consists of two quarter circles and one line segment, as shown in the figure above. Let  $g(x) = 2x + \int_0^x f(t) dt$ .

(a) Find  $g(-3)$ . Find  $g'(x)$  and evaluate  $g'(-3)$ .

(b) Determine the  $x$ -coordinate of the point at which  $g$  has an absolute maximum on the interval  $-4 \leq x \leq 3$ . Justify your answer.

(c) Find all values of  $x$  on the interval  $-4 < x < 3$  for which the graph of  $g$  has a point of inflection. Give a reason for your answer.

(d) Find the average rate of change of  $f$  on the interval  $-4 \leq x \leq 3$ . There is no point  $c$ ,  $-4 < c < 3$ , for which  $f'(c)$  is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

$$g'(x) = 2 + f(x)$$

$$g''(x) = f'(x)$$

(b)  $g'(x) = 0$

$$0 = 2 + f(x) \rightarrow f(x) = -2$$

$x$	$g'(x)$
-4	not
2.5	Necessary
3	this time

$$g'(x) > 0 \quad (-4, 2.5)$$

$$g'(x) < 0 \quad (2.5, 3)$$

SO the abs max of  $g$  is at  $x=2.5$