(a) $W'(12) \approx \frac{W(15)-W(9)}{15-9} = \frac{67.9-61.8}{15-9} = \frac{6.1}{6}$

The temp of H_2O is increasing at a rate of $\frac{6.1}{6}$ °F/min at $t=12$

(b) $\int_0^{20} W'(t)\,dt = W(t)\Big|_0^{20} = W(20) - W(0) = 71 - 55 = 16°F$

over the 1st 20 min → the temp increared 16°F

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

   (a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

   (b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

   (c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

   (d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

(c) $\frac{1}{20}\left(4(55) + 5(57.1) + 6(61.8) + 5(67.9)\right)$ °F

underestimate W(t) is increasing and we used a Left Riemann sum

(d) $W(25) = W(20) + \int_{20}^{25} W'(t)\,dt = 73.043°F$

4. The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$. $f(x) = (25-x^2)^{\frac{1}{2}}$

(a) Find $f'(x)$. $f'(x) = \frac{1}{2}(25-x^2)^{-\frac{1}{2}}(-2x)$

(b) Write an equation for the line tangent to the graph of f at $x = -3$.

$f(-3) = \sqrt{25-(-3)^2} = \sqrt{25-9} = \sqrt{16} = 4$ $\quad f'(-3) = \frac{1}{2}(25-9)^{-\frac{1}{2}}(-2\cdot -3) = +\frac{3}{4}$

(c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \le x \le -3 \\ x + 7 & \text{for } -3 < x \le 5. \end{cases}$

Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

(d) Find the value of $\int_0^5 x\sqrt{25 - x^2}\, dx$.

(b) $y - 4 = \frac{3}{4}(x+3)$

(c) $\lim_{x \to -3^-} g(x) = 4$

$\lim_{x \to -3^+} g(x) = -3+7 = 4$

$g(-3) = 4$

yes continuous at $x = -3$

(d) $\int_0^5 x\sqrt{25-x^2}\, dx$

$u = 25 - x^2$

$\frac{du}{dx} = -2x$

$-\frac{1}{2}du = x\,dx$

$-\frac{1}{2}\int_{25}^{0} u^{\frac{1}{2}}\, du$

$u = 25 - 5^2 = 0$

$u = 25 - 0^2 = 25$

$-\frac{1}{2}\cdot\frac{2}{3}u^{\frac{3}{2}}\Big|_{25}^{0}$ $\boxed{= -\frac{1}{3}(0)^{3/2} + \frac{1}{3}(25)^{3/2}}$

GO ON TO THE NEXT PAGE.

6. For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

   (a) For $0 \le t \le 12$, when is the particle moving to the left? $v(t) < 0$ $\quad t \in (3, 9)$ $\quad 3 < t < 9$

   (b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$. $\int_0^6 |v(t)|\,dt$ or $\int_0^3 v(t)\,dt - \int_3^6 v(t)\,dt$

   (c) Find the acceleration of the particle at time t. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning. $a(t) = v'(t) = -\sin\left(\frac{\pi}{6}t\right)\frac{\pi}{6}$ $\quad a(4) = -\sin\left(\frac{4\pi}{6}\right)\frac{\pi}{6}$

   (d) Find the position of the particle at time $t = 4$.

$a(4) < 0$

(d) $x(4) = x(0) + \int_0^4 v(t)\,dt$

(a) $\cos\left(\frac{\pi}{6}t\right) = 0$ $\quad \frac{6}{\pi} \cdot \frac{\pi}{6}t = \left(\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \text{etc.}\right)\frac{6}{\pi}$

$t = 3, 9$

$v(t)$: $+$ on $(0,3)$, $-$ on $(3,9)$, $+$ on $(9,12)$

0 3 9 12

$v(4) = \cos\left(\frac{4\pi}{6}\right)$

$v(4) < 0$

speed at $t=4$ is increasing

$a(t)$ & $v(t)$ are both negative

STOP

END OF EXAM

$$-2 + \int_0^4 \cos\left(\frac{\pi}{6}t\right)dt$$

$$-2 + \frac{6}{\pi}\sin\left(\frac{\pi}{6}t\right)\Big|_0^4$$

$$\boxed{-2 + \frac{6}{\pi}\sin\left(\frac{4\pi}{6}\right) - \frac{6}{\pi}\sin(0)}$$

$$-2 + \frac{6}{\pi}\cdot\frac{\sqrt{3}}{2} - 0 = \boxed{-2 + \frac{3\sqrt{3}}{\pi}}$$