

NO CALCULATOR ALLOWED

6. Let f be a function with $f(2) = -8$ such that for all points (x, y) on the graph of f , the slope is given by $\frac{3x^2}{y}$.

(a) Write an equation of the line tangent to the graph of f at the point where $x = 2$ and use it to approximate $f(1.8)$.

$$2, f(2) = -8, f'(2) = \frac{3(2)^2}{-8} = -\frac{3}{2}$$

$$y + 8 = -\frac{3}{2}(x - 2)$$

$$f(1.8) \approx -\frac{3}{2}(1.8 - 2) - 8 = \boxed{-7.7}$$

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(b) Find an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = \frac{3x^2}{y}$ with the initial condition

$$f(2) = -8$$

$$\int y \, dy = \int 3x^2 \, dx$$

$$\frac{1}{2}y^2 = x^3 + C$$

$$\frac{1}{2}(-8)^2 = (2)^3 + C$$

$$32 = 8 + C$$

$$C = 24$$

$$\frac{1}{2}y^2 = x^3 + 24$$

$$\sqrt{y^2} = \sqrt{2x^3 + 48}$$

$$y = -\sqrt{2x^3 + 48}$$

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