Free Response Section 2: Non Calculator

t (days)	0	10	22	30
$W'(t)$ (GL per day)	0.6	0.7	1.0	0.5

3. The twice-differentiable function W models the volume of water in a reservoir at time t, where $W(t)$ is measured in gigaliters (GL) and t is measured in days. The table above gives values of $W'(t)$ sampled at various times during the time interval $0 \le t \le 30$ days. At time $t = 30$, the reservoir contains 125 gigaliters of water.

(a) Use the tangent line approximation to W at time $t = 30$ to predict the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 32$. Show the computations that lead to your answer.

$y - y_1 = m(x - x_1)$

$x_1 = 30$

$y_1 = 125$

$m = w'(30) = .5$

$y - 125 = (.5)(t - 30)$

$\boxed{W(32) \approx (.5)(32-30) + 125 \text{ gL}}$

(b) Use a left Riemann sum, with the three subintervals indicated by the data in the table, to approximate $\int_0^{30} W'(t)\,dt$. Use this approximation to estimate the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 0$. Show the computations that lead to your answer.

$(10)(.6) + (12)(.7) + (8)(1) \approx W(30) - w(0)$

$6 + 8.4 + 8 = 22.4 \approx w(30) - w(0)$

$W(0) + \int_0^{30} w'(t)\,dt = w(30)$ $W(0) + 22.4 = 125$ $\boxed{W(0) \approx 125 - 22.4}$

(c) Explain why there must be at least one time t, other than $t = 10$, such that $W'(t) = 0.7$ GL/day.

W is twice-differentiable therefore $W'(t)$ a continuous function

$w'(22) = 1$

$w'(30) = .5$

$.5 < .7 < 1$

by Intermediate Value Theorem

(d) The equation $A = 0.3W^{2/3}$ gives the relationship between the area A, in square kilometers, of the surface of the reservoir, and the volume of water $W(t)$, in gigaliters, in the reservoir. Find the instantaneous rate of change of A, in square kilometers per day, with respect to t when $t = 30$ days.

$\frac{dA}{dt} = \frac{dA}{dW}\frac{dW}{dt}$

$\left.\frac{dW}{dt}\right|_{t=30} = .5$

$\frac{dA}{dW} = \frac{2}{3}(.3)W^{-\frac{1}{3}}$

$\boxed{\left.\frac{dA}{dt}\right|_{t=30} = \frac{2}{3}(.3)(125)^{-\frac{1}{3}}(.5)}$