(b)

$f''(c) = \frac{f'(b) - f'(a)}{b-a}$

$0 = \frac{f'(1) - f'(-1)}{1 - -1}$

$= \frac{0-0}{2}$

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

5. The twice-differentiable functions f and g are defined for all real numbers x. Values of f, f', g, and g' for various values of x are given in the table above.

(a) Find the x-coordinate of each relative minimum of f on the interval $[-2, 3]$. Justify your answers.

at $x=1$ since $f'(x)=0$ and changes $-$ to $+$

(b) Explain why there must be a value c, for $-1 < c < 1$, such that $f''(c) = 0$.

MVT, f is twice-differentiable

(c) The function h is defined by $h(x) = \ln(f(x))$. Find $h'(3)$. Show the computations that lead to your answer.

$h'(x) = \frac{1}{f(x)} f'(x)$ $\qquad h'(3) = \frac{f'(3)}{f(3)} = \frac{1/2}{7}$ or $1/14$

(d) Evaluate $\int_{-2}^{3} f'(g(x))g'(x)\,dx$.

$u = g(x)$ $\qquad u_T = g(3) = 1$

$\frac{du}{dx} = g'(x)$ $\qquad \int_{-1}^{1} f'(u)\,du$ $\qquad u_B = g(-2) = -1$

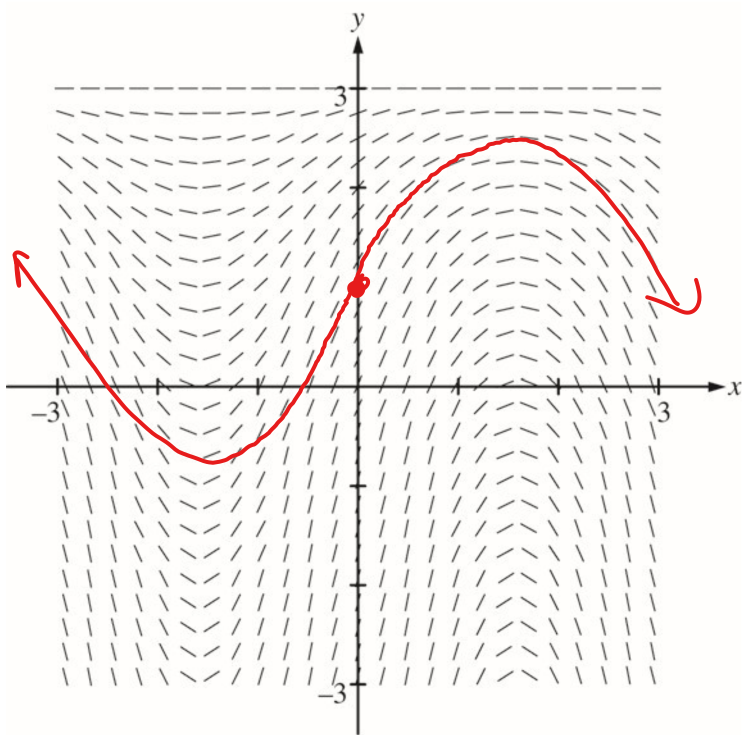
$du = g'(x)\,dx$

$f(u)\Big|_{-1}^{1} = f(1) - f(-1)$

$2 - 8 = -6$

6. Consider the differential equation $\frac{dy}{dx} = (3 - y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers.

   (a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0, 1)$.



   (b) Write an equation for the line tangent to the solution curve in part (a) at the point $(0, 1)$. Use the equation to approximate $f(0.2)$. $\left.\frac{dy}{dx}\right|_{0,1} = (3-1)\cdot\cos(0) = 2$ $\quad y-1 = 2(x-0)$

   (c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$.

$\boxed{f(2) \approx 2(2-0)+1}$

$$\frac{dy}{dx} = (3-y)\cos x$$

$$\int \frac{1}{3-y}\,dy = \int \cos x\,dx$$

$$\frac{1}{-1}\ln|3-y| = \sin x + c$$

$$-\ln|3-1| = \sin(0) + c$$

$$-\ln 2 = c$$

$$-\ln|3-y| = \sin x - \ln 2$$

$$e^{\ln|3-y|} = e^{-\sin x + \ln 2}$$

$$3-y = 2e^{-\sin x}$$

$$-y = 2e^{-\sin x} - 3$$

$$\boxed{y = -2e^{-\sin x} + 3}$$