NO CALCULATOR ALLOWED

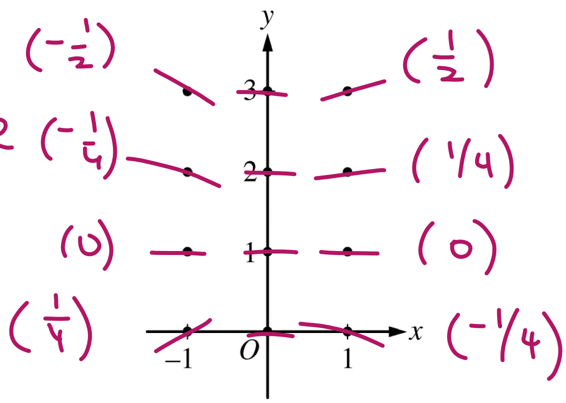
4. Consider the differential equation $\frac{dy}{dx} = \frac{x(y-1)}{4}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

$y-1=0$ if $y=1$

$x=0 \rightarrow 0$ slope

(y axis



(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 3$. Write an equation for the line tangent to the graph of f at the point $(1, 3)$ and use it to approximate $f(1.4)$.

$\left.\frac{dy}{dx}\right|_{(1,3)} = \frac{(1)(3-1)}{4} = \frac{1}{2}$

$y - 3 = \frac{1}{2}(x-1)$

$f(1.4) \approx \frac{1}{2}(1.4-1) + 3$

Continue problem 4 on page 17.

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$\frac{dy}{dx} = \frac{x(y-1)}{4}$

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = 3$.

$\int \frac{1}{y-1}\,dy = \int \frac{1}{4}x\,dx$

$\ln|y-1| = \frac{1}{8}x^2 + c$

$\ln|3-1| = \frac{1}{8} + c$

$\ln 2 = \frac{1}{8} + c$

$\ln 2 - \frac{1}{8} = c$

$\ln|y-1| = \frac{1}{8}x^2 + \ln 2 - \frac{1}{8}$

$e^{\wedge}$ $e^{\wedge}$

$y-1 = 2e^{\frac{1}{8}x^2 - \frac{1}{8}}$

$y = 2e^{\frac{1}{8}x^2 - \frac{1}{8}} + 1$

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