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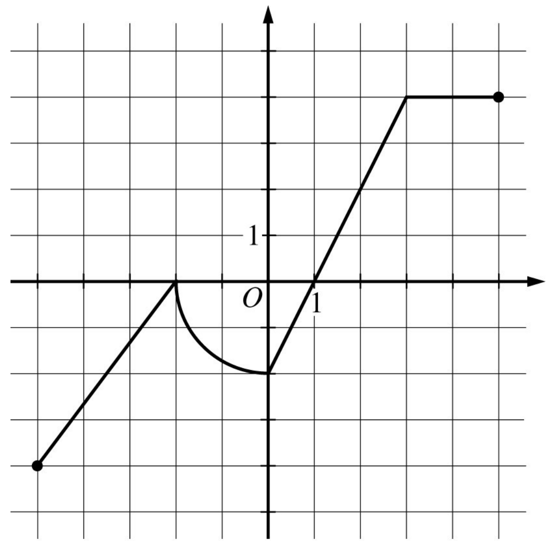

Graph of f

3. The graph of the function f, consisting of three line segments and a quarter of a circle, is shown above. Let g be the function defined by $g(x) = \int_1^x f(t)\,dt$.

$g(5) = \int_1^5 f(t)\,dt = 12$

$g(-5) = \int_1^{-5} f(t)\,dt = 6 + \pi + 1$

(a) Find the average rate of change of g from $x = -5$ to $x = 5$.

$$\frac{g(5) - g(-5)}{5 - -5} = \frac{12 - (6 + \pi + 1)}{10}$$

$$\frac{12 - 6 - \pi - 1}{10} = \boxed{\frac{5 - \pi}{10}}$$

(b) Find the instantaneous rate of change of g with respect to x at $x = 3$, or state that it does not exist.

$g'(x) = f(x)$

$g'(3) = f(3) = 4$

Continue question 3 on page 15.

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(c) On what open intervals, if any, is the graph of g concave up? Justify your answer.

$g''(x) = f'(x)$ $(-5, -2)$ $(0, 3)$

$f'(x) > 0$

(d) Find all x-values in the interval $-5 < x < 5$ at which g has a critical point. Classify each critical point as the location of a local minimum, a local maximum, or neither. Justify your answers.

open

g has critical pts
when $g' = 0$ or g' is undefined

at $x = -2$ neither because f does not change signs

at $x = 1$ is location of local minimum f changes neg to pos

GO ON TO THE NEXT PAGE.

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$\frac{6-0}{3} = 2$

x	0	1	2	3	4	5	6
$f'(x)$	4	3.5	2	0.8	1.7	5.8	7

4. The function f satisfies $f(0) = 20$. The first derivative of f satisfies the inequality $0 \le f'(x) \le 7$ for all x in the closed interval $[0, 6]$. Selected values of f' are shown in the table above. The function f has a continuous second derivative for all real numbers.

(a) Use a midpoint Riemann sum with three subintervals of equal length indicated by the data in the table to approximate the value of $f(6)$.

$f(6) = f(0) + \int_0^6 f'(x)\,dx$

$= 20 + 2(3.5) + 2(8) + 2(5.8)$

$20 + 7 + 16 + 11.6$

$27 + 13.2$

40.2

(b) Determine whether the actual value of $f(6)$ could be 70. Explain your reasoning.

$f(0) + \int_0^6 f'(x)\,dx$

$20 + \int_0^6 7\,dx$

$20 + 7x\Big|_0^6$

$7(6) - 7(0)$

$20 + 42 = 62$ Not possible for $f(6) = 70$

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Continue question 4 on page 17.

(c) Evaluate $\int_2^4 f''(x)\,dx$.

$f'(4) - f'(2)$

$1.7 - 2$

-3

(d) Find $\lim_{x\to 0} \dfrac{f(x) - 20e^x}{0.5f(x) - 10}$ $= \lim_{x\to 0} \dfrac{f'(x) - 20e^x}{5f'(x)} = \dfrac{4-20}{5(4)} = \dfrac{-16}{2} = -8$

$\lim_{x\to 0} f(x) - 20e^x = 20 - 20 = 0$

$\lim_{x\to 0} 5f(x) - 10 = 10 - 10 = 0$

L'Hospitals

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