Precalculus   Name_____

Solving Triangles and Area Formula Notes   Date_____ Period_____

Solve each triangle. Round your answers to the nearest tenth.

1) In $\triangle PKH$, $h = 18$ yd, $m\angle P = 142°$, $k = 25$ yd

$$p^2 = 18^2 + 25^2 - 2(18)(25)\cos 142°$$

$\boxed{p = 40.7 \text{ yd}}$

$$\frac{\sin 142°}{40.7} = \frac{\sin H}{18} \rightarrow \sin^{-1}\left(\frac{18\sin 142°}{40.7}\right) = \boxed{\angle H = 15.8°}$$

$$180 - 15.8 - 142 = \boxed{\angle K = 22.2°}$$

2) In $\triangle FDE$, $m\angle F = 50°$, $e = 32$ km, $f = 31$ km

$$\frac{\sin 50°}{31} = \frac{\sin E}{32}$$

$$\frac{32\sin 50°}{31} = \sin E$$

$$\angle E = \sin^{-1}\left(\frac{32\sin 50°}{31}\right)$$

$\boxed{\angle E = 52.3°}$

and $\triangle$?

$180° - 52.3° = \boxed{127.7° = \angle E}$ 2nd $\triangle$

$$\boxed{\angle D = \begin{matrix} 180° \\ -50° \\ -52.3° \\ \hline 77.7° \end{matrix}}$$

or $\boxed{\angle D \text{ (2nd)} = \begin{matrix} 180° \\ -50° \\ -127.7° \\ \hline 2.3° \end{matrix}}$

$$\frac{\sin 77.7°}{d} = \frac{\sin 50°}{31}$$ or $$\frac{\sin 2.3°}{d} = \frac{\sin 50°}{31}$$

$$\boxed{d = \frac{31\sin 77.7°}{\sin 50°} = 39.5 \text{ km}}$$ or $$\boxed{d \text{ (2nd)} = \frac{31\sin 2.3°}{\sin 50°} = 1.6 \text{ km}}$$

Find the area of each triangle to the nearest tenth. $A = \frac{1}{2}bh$ is weak $\longrightarrow$ $A = \frac{1}{2}ab\sin C \rightarrow$ better

3)

R
$67°$
5 cm
Q $26°$ $87°$ P
10.5 cm
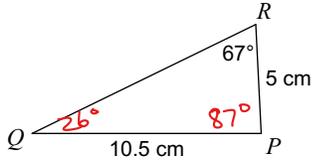
$\dfrac{\sin 67°}{10.5} = \dfrac{\sin Q}{5}$

$\dfrac{5\sin 67°}{10.5} = \sin Q$

$\sin^{-1}\left(\dfrac{5\sin 67°}{10.5}\right) = Q = 26°$

$A = \frac{1}{2}(5)(10.5)\sin 87°$

$\boxed{= 26.2 \text{ cm}^2}$

4)
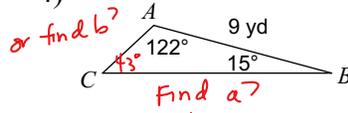
or find b?
A 9 yd
$43°$ $122°$ $15°$
C B
Find a?

$\downarrow$
we'll do this (but it doesn't matter)

$\dfrac{\sin 122°}{a} = \dfrac{\sin 43°}{9}$

$a = \dfrac{9\sin 122°}{\sin 43°} = 11.2$

$A = \frac{1}{2}(11.2)(9)\sin 15° = \boxed{13 \text{ yd}^2}$

another option if you have SSS $\rightarrow$ $\left(S = \text{Semiperimeter} = \dfrac{\text{Perimeter}}{2}\right)$

$A = \sqrt{S(S-a)(S-b)(S-c)} = \sqrt{(12.75)(12.75-8.4)(12.75-11)(12.75-6.1)} = 25.4 \text{ km}^2$

$S = \left(\dfrac{8.4}{+6.1}\atop{+11}\right) \div 2$

$= 12.75$

5)

S 8.4 km R
$97.4°$
6.1 km
11 km
T

$A = \frac{1}{2}(8.4)(6.1)\sin 97.4°$

$= 25.4 \text{ km}^2$

$11^2 = 6.1^2 + 8.4^2 - 2(6.1)(8.4)\cos S$

$\dfrac{11^2 - 6.1^2 - 8.4^2}{(-2)(6.1)(8.4)} = \cos S$

$\cos^{-1}\left(\dfrac{11^2 - 6.1^2 - 8.4^2}{(-2)(6.1)(8.4)}\right) = \angle S$

6)

R
$117°$ 6.1 $180 - (117 + 33) = 30°$
S $33°$ T
10 ft
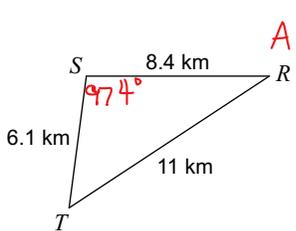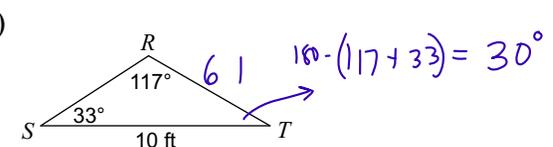
$\dfrac{\sin 117°}{10} = \dfrac{\sin 33°}{S}$

$S = \dfrac{10\sin 33°}{\sin 117°} = 6.1$

$A = \frac{1}{2}(10)(6.1)\sin 30°$

$\boxed{A = 15.3 \text{ ft}^2}$

7)

H, 11 mi, K, 5 mi, 46°, P
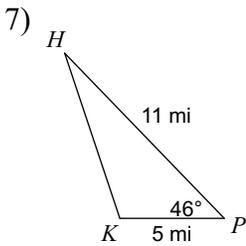
$A = \frac{1}{2}(11)(5)\sin 46°$

$= 19.8 \text{ mi}^2$

8) In $\triangle DEF$, $f = 6$ m, $d = 8$ m, $m\angle D = 47°$

D, 47°, 6, E, 8, F = 33.3°

$\begin{array}{r} 33.3 \\ +47.0 \\ \hline 80.3 \end{array}$
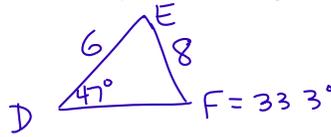
$\frac{\sin 47°}{8} = \frac{\sin F}{6}$

$\frac{6\sin 47°}{8} = \sin F$

$F = \sin^{-1}\left(\frac{6\sin 47°}{8}\right) = 33.3°$

$\begin{array}{r} 180° \\ -80.3° \\ \hline 99.7° = \angle E \end{array}$

$A = \frac{1}{2}(6)(8)\sin 99.7°$

$= 23.7 \text{ m}^2$

9) Assuming the area of $\triangle KHP$ is 59.3 cm^2, $p = 9$ cm, $k = 15$ cm and $\angle H$ is obtuse, find the measure of $\angle P$.

(1)

H, 9, 15, k, P
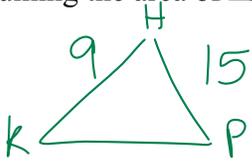
First answer ↓

$\angle H = 118.54°$

② now L O C
$h^2 = 15^2 + 9^2 - 2(15)(9)\cos(118.54°)$

$h = 20.9$ cm

③ L O S
$\frac{\sin 118.54°}{20.9} = \frac{\sin P}{9}$

$\sin^{-1}\left(\frac{9\sin 118.54°}{20.9}\right) = \boxed{P = 22.3°}$

$A = \frac{1}{2}ab\sin C$

$59.3 = \frac{1}{2}(9)(15)\sin H$

$\frac{2(59.3)}{(9)(15)} = \sin H$

$\angle H = \sin^{-1}\left(\frac{(2)(59.3)}{(9)(15)}\right) = 61.46$ but obtuse, so really $180 - 61.46$

$= 118.54°$

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 $m\angle K = 22.2°, m\angle H = 15.8°, p = 40.7$ yd

2) In $\triangle FDE$, $m\angle F = 50°$, $e = 32$ km, $f = 31$ km

 $m\angle D = 77.7°, m\angle E = 52.3°, d = 39.5$ km
 Or $m\angle D = 2.3°, m\angle E = 127.7°, d = 1.6$ km

Find the area of each triangle to the nearest tenth.

3)
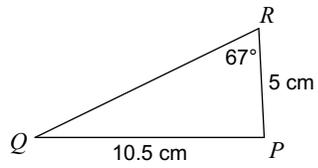
26.2 cm²

4)
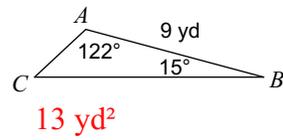
13 yd²

5)

25.4 km²

6)

15.2 ft²

7)

H

11 mi

46°

K 5 mi P

19.8 mi²
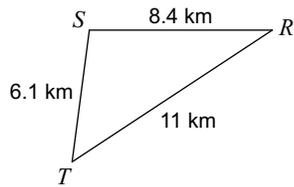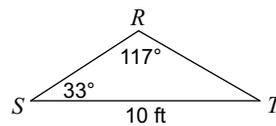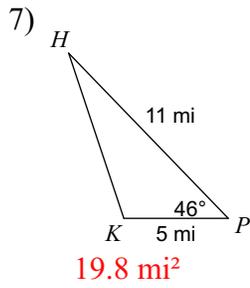
8) In $\triangle DEF, f = 6$ m, $d = 8$ m, $m\angle D = 47°$

23.7 m²

9) Assuming the area of $\triangle KHP$ is 59.3 cm^2, $p = 9$ cm, $k = 15$ cm and $\angle H$ is obtuse, find the measure of $\angle P$.

22.3°