AP Calculus
3.5 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Find the following derivatives … AND MEMORIZE THEM ASAP!

a) $\frac{d}{dx}[\sin x]$ $\cos x$ b) $\frac{d}{dx}[\cos x]$ $-\sin x$ c) $\frac{d}{dx}[\tan x]$ $\sec^2 x$

d) $\frac{d}{dx}[\sec x]$ $\sec x \tan x$ e) $\frac{d}{dx}[\csc x]$ $-\csc x \cot x$ f) $\frac{d}{dx}[\cot x]$ $-\csc^2 x$

2. Find $\frac{dy}{dx}$ for each of the following:

a) $y = 3 - x - \tan x$

$\frac{dy}{dx} = -1 - \sec^2 x$

b) $y = x \csc x$

$\frac{dy}{dx} = 1 \cdot \csc x + x(-\csc x \cot x)$

c) $y = \frac{1}{x} + 7x^2 \sin x$ ($\frac{1}{x} \to x^{-1}$)

$\frac{dy}{dx} = -\frac{1}{x^2} + 14x \sin x + 7x^2 \cos x$

d) $y = \frac{\cot x}{5 - \cos x}$ $\frac{dy}{dx} = \frac{(5-\cos x)(-\csc^2 x) - (\cot x)(\sin x)}{(5-\cos x)^2}$

3. If $y = \tan x - \cot x$, then $\frac{dy}{dx} =$

$\frac{dy}{dx} = \sec^2 x + \csc^2 x$

4. If $f(x) = \frac{x}{\tan x}$, then $f'\left(\frac{\pi}{4}\right) =$

$f'(x) = \frac{\tan x (1) - x(\sec^2 x)}{\tan^2 x}$

$f'(\pi/4) = \frac{(1)(1) - \frac{\pi}{4}(\sqrt{2})^2}{1^2}$

$= 1 - \frac{2\pi}{4} = 1 - \frac{\pi}{2}$

$y' = \frac{dy}{d\theta}$

5. If $y = \sec x$, find $\frac{d^2y}{dx^2}$.

$\frac{dy}{dx} = \sec x \tan x$

$\frac{d^2y}{dx^2} = \sec x \tan x \tan x + \sec x \sec^2 x$

6. If $y = \theta \tan \theta$, find y''.

$y' = \theta \sec^2\theta + 1 \cdot \tan\theta$ ✓

$y' = \theta(\sec\theta)^2 + \tan\theta$

$y'' = \theta \cdot 2\sec\theta \cdot \sec\theta\tan\theta + 1\cdot(\sec\theta)^2 + \sec^2\theta$

7. If $f(x) = \sin x$, find $f'(x)$, $f''(x)$, $f'''(x)$, and $f^{(4)}(x)$. What do you think the function $f^{(100)}(x)$ is?

$f'(x) = \cos x$ $\quad f'''(x) = -\cos x$ $\quad f^{100}(x) = \sin x$

$f''(x) = -\sin x$ $\quad f^4(x) = \sin x$

8. Find an equation of the tangent line and the normal line to the graph of $y = x + \cos x$ at the point (0, 1).

$y' = 1 - \sin x$

$1 - \sin x \Rightarrow 1 - \sin 0 = 1$

T: $y - 1 = 1(x - 0)$

N: $y - 1 = -1(x - 0)$

9. [Calculator] Find equations for the lines that are tangent and normal to the curve $y = x^3 \sin x$ when $x = 2$.

$y' = 3x^2\sin x + x^3\cos x$

$y = 8\sin 2$ when $x = 2$

$y'(2) = 7.582$

T: $y - 8\sin 2 = 7.582(x - 2)$

N: $y - 8\sin 2 = -.132(x - 2)$

10. Find the points on the curve $y = \cot x$, $0 < x < \pi$, where the tangent line is parallel to the line $y = -2x$.

$y = -\csc^2 x$

Slope = $\nearrow -2$

$-\csc^2 x = -2$

$\csc^2 x = 2$ $\quad \sin^2 x = \frac{1}{2}$

$\sin x = \pm\frac{\sqrt{2}}{2}$

$x = \left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$

Pts: $\left(\frac{\pi}{4}, 1\right)$ $\left(\frac{3\pi}{4}, -1\right)$

11. $\lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h} =$

$\frac{d}{dx}\sin x = \cos x$

A) 0
B) 1
C) $\sin x$
D) $\cos x$
E) nonexistent