Name____ Date________________________ Period______

Worksheet 6.2—Definite Integrals & Numeric Integration
Show all work on a separate sheet of paper.

Multiple Choice

1. (Calculator Permitted) If the midpoints of 4 equal-width rectangles is used to approximate the area enclosed between the *x*-axis and the graph of $y = 4x - x^2$, the approximation is

   (A) 10 (B) 10.5 (C) 10.666 (D) 10.75 (E) 11

2. If $\int_2^5 f(x)dx = 18$, then $\int_2^5 \left(f(x)+4\right)dx =$

   (A) 20 (B) 22 (C) 23 (D) 25 (E) 30

3. $\int_{-4}^{4} \left(4-|x|\right)dx =$

   (A) 0 (B) 4 (C) 8 (D) 16 (E) 32

4. If $\int_a^b f(x)dx = a+2b$, then $\int_a^b \left(f(x)+3\right)dx =$

   (A) $a+2b+3$ (B) $3b-3a$ (C) $4a-b$ (D) $5b-2a$ (E) $5b-3a$

5. The expression $\frac{1}{20}\left(\sqrt{\frac{1}{20}}+\sqrt{\frac{2}{20}}+\sqrt{\frac{3}{20}}+\ldots+\sqrt{\frac{20}{20}}\right)$ is a Riemann sum approximation for

   (A) $\int_0^1 \sqrt{\frac{x}{20}}dx$ (B) $\int_0^1 \sqrt{x}dx$ (C) $\frac{1}{20}\int_0^1 \sqrt{\frac{x}{20}}dx$ (D) $\frac{1}{20}\int_0^1 \sqrt{x}dx$ (E) $\frac{1}{20}\int_0^{20} \sqrt{x}dx$

Short Answer

6. The table below gives the values of a function obtained from an experiment. Use them to estimate $\int_0^6 f(x)dx$ using **three equal subintervals** with a) right endpoints, b) left endpoints, c) midpoints, and d) the trapezoidal rule. If the function is said to be a decreasing function, can you say whether your estimates are less than or greater than the exact value of the integral? Could any of these estimates approximate the area of the enclosed region with the *x*-axis? Why or why not?

x	0	1	2	3	4	5	6
$f(x)$	9.3	9.0	8.3	6.5	2.3	−7.6	−10.5

$\frac{\pi - 0}{3} = \frac{\pi}{3}$

x	0	$\pi/3$	$2\pi/3$	π
$\sin x$	0	$\sqrt{3}/2$	$\sqrt{3}/2$	0

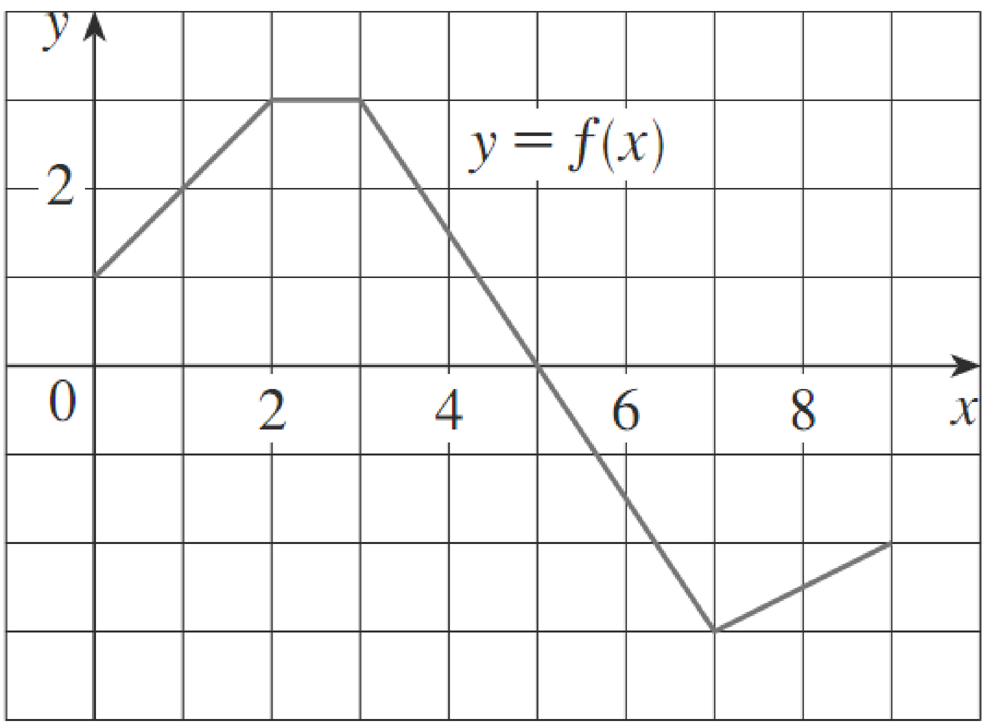
x	$\pi/6$	$3\pi/6$	$5\pi/6$
$\sin x$	$1/2$	1	$1/2$

7. Approximate the area of the region bounded by the graph of $y = \sin x$ and the x-axis from $x = 0$ to $x = \pi$ using 3 equal subintervals using a) left endpoints, b) right endpoints, c) midpoints, and d) trapezoidal rule.

a, b, d $\rightarrow \frac{\pi}{3}\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + 0\right) = \frac{\pi\sqrt{3}}{3}$ MRAM $= \frac{\pi}{3}\left(\frac{1}{2} + 1 + \frac{1}{2}\right) = \frac{2\pi}{3}$

8. The graph of f is shown below. Evaluate each integral by interpreting it in terms of areas.

a) $\int_0^2 f(x)\,dx$ 4 b) $\int_0^5 f(x)\,dx$ 10 c) $\int_5^7 f(x)\,dx$ -3 d) $\int_0^9 f(x)\,dx$ 2



9. Find $\int_{\boxed{0}}^{\boxed{5}} f(x)\,dx$ if $f(x) = \begin{cases} 3, & \boxed{x < 3} \\ x, & \boxed{x \ge 3} \end{cases}$ 17

$\int_0^3 3\,dx + \int_3^5 x\,dx$

$3x\Big|_0^3 = 9$ $\frac{1}{2}x^2\Big|_3^5$

$\frac{1}{2}(5)^2 - \frac{1}{2}(3)^2$

$\frac{25}{2} - \frac{9}{2} = \frac{16}{2} = 8$

10. Given that $\int_4^9 \sqrt{x}\,dx = \frac{38}{3}$, what is

$38 + 45 \Rightarrow 83$

a) $\int_9^4 \sqrt{t}\,dt$ $-\frac{38}{3}$ b) $\int_4^9 \left(\sqrt{x} + 3\right)dx$ $\frac{83}{3}$ c) $\int_9^{14} \sqrt{x - 5}\,dx$ $\frac{38}{3}$ d) $\int_4^4 \sqrt{x}\,dx$ 0

$+\frac{38}{3} + 15$

11. If $f(x)$ is represented by the table below, approximate $\int_1^{9.6} f(x)\,dx$ using left-endpoint, right-endpoint, midpoint, and trapezoidal approximations. Use as many subintervals as the data allows.

x	1	2.5	4	6	8	8.8	9.6	~~10.4~~
$f(x)$	4	3	1	3	5	6	4	~~7~~

L, R, T $\rightarrow$ only use 6

Midpoint $\rightarrow$ only use 3

LRAM $= (1.5)(4) + (1.5)(3) + (2)(1) + (2)(3) + (.8)(5) + (.8)(6) = 27.3$

RRAM $= (1.5)(3) + (1.5)(1) + (2)(3) + (2)(5) + (.8)(6) + (.8)(4) = 30$

Trap $= \frac{\text{RRAM} + \text{LRAM}}{2} = 28.65$

MRAM $= (3)(3) + (4)(3) + (1.6)(6) = 30.6$

12. Write as a single integral in the form $\int_a^b f(x)dx$: $\int_{-2}^{2} f(x)dx + \int_2^5 f(x)dx - \int_{-2}^{-1} f(x)dx$

13. If $\int_1^5 f(x)dx = 12$ and $\int_4^5 f(x)dx = 3.6$, find $\int_1^4 2f(x)dx$

14. If $\int_0^9 f(x)dx = 37$ and $\int_0^9 g(x)dx = 16$, find $\int_0^9 \left[2f(x) + 3g(x)\right]$

15. (Calculator Permitted) Use your calculator's fnInt(function to evaluate the following integrals. Report 3 decimials.

a) $\int_0^5 \frac{x}{x^2+4}dx$

b) $3 + 2\int_0^{\pi/3} \tan x dx$