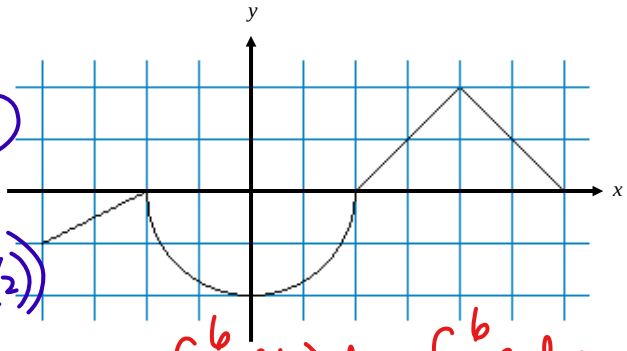
AP Calculus
5.3 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. The graph of f below consists of line segments and a semicircle. Evaluate each definite integral.



a) $\int_0^2 f(x)\,dx$ $\frac{-\pi(2)^2}{4} = -\pi$

b) $\int_2^6 f(x)\,dx$ $\frac{1}{2}(4)(2) = 4$

c) $\int_{-4}^2 f(x)\,dx$ $-\frac{1}{2}(2)(1) - \frac{1}{2}\pi(2)^2 = -1-2\pi$

d) $\int_4^0 f(x)\,dx$ $-\left(-\pi + \frac{1}{2}(2)(2)\right) = \pi - 2$

e) $\int_{-4}^6 |f(x)|\,dx$ $\frac{1}{2}(2)(1) + \frac{1}{2}(\pi)(2)^2 + \frac{1}{2}(4)(2) = 1+2\pi+4 = 5+2\pi$

f) $\int_{-4}^6 [f(x)+2]\,dx$ $\int_{-4}^6 f(x)\,dx + \int_{-4}^6 2\,dx$ $-1-2\pi+4+20 = 23-2\pi$

2. Part *e* above, gives a way to find the total <u>area</u> between the *x*–axis and the function between $x = -4$ and $x = 6$. *Without using absolute value signs*, write an expression that can be used to find the total <u>area</u> between the *x*–axis and the function between $x = -4$ and $x = 6$.

$\int_2^{-4} f(x)\,dx + \int_2^6 f(x)\,dx$

$\int_1^2 f(x)\,dx + \int_2^5 f(x)\,dx = \int_1^5 f(x)\,dx$

3. Suppose that f and g are continuous and $\int_1^2 f(x)\,dx = -4$, $\int_1^5 f(x)\,dx = 6$, and $\int_1^5 g(x)\,dx = 8$.

Find each of the following:

a) $\int_2^2 g(x)\,dx$ 0

b) $\int_5^1 7g(x)\,dx$ $-7(8) = -56$

c) $\int_1^2 3f(x)\,dx$ $3(-4) = -12$

d) $\int_2^5 f(x)\,dx$ $6-(-4) = 10$

e) $\int_1^5 [f(x)-g(x)]\,dx$ $6-8 = -2$

f) $\int_1^5 [9f(x)+4]\,dx$ $9\int_1^5 f(x)\,dx + \int_1^5 4\,dx$ $96+44 = 70$

4. What are all the values of *k* for which $\int_2^k x^2\,dx = 0$?

A –2
B 0
C 2
D –2 and 2
E –2, 0, and 2

5. If $\int_3^7 f(x)\,dx = 5$ and $\int_3^7 g(x)\,dx = 3$, then all of the following must be true *except*

A $\int_3^7 f(x)g(x)\,dx = 15$ (NO multiplication rule)

B $\int_3^7 [f(x)+g(x)]\,dx = 8$

C $\int_3^7 2f(x)\,dx = 10$

D $\int_3^7 [f(x)-g(x)]\,dx = 2$

E $\int_7^3 [g(x)-f(x)]\,dx = 2$

6. A driver average 30 mph on a 150-mile trip and then returned over the same 150 miles at the rate of 50 mph. He figured his average speed was 40 mph for the entire trip.

a) What was the total distance traveled?

300 miles

b) What was his total time spent for the trip?

5 hr there + 3 hr back → 8 hr

c) What was his average speed for the trip?

$\frac{300 \text{ miles}}{8 \text{ hr}} = 37.5 \text{ mph}$

d) Explain the driver's error in reasoning.

→ forgot that the average should be closer to 30 mph since that occured over a longer time

7. A dam released 1000 m^3 of water at 10 m^3/min and then released another 1000 m^3 at 20 m^3/min. What was the average rate at which the water was released? Give reasons for your answer.

1000 + 1000 = 2000

$\frac{1000}{10} = 100$ min $\quad \frac{1000}{20} = 50$ min

$\frac{2000 m^3}{150 \text{ min}} = \frac{40}{3}$ or $13\frac{1}{3}$ m^3/min

8. [Calculator] At different altitudes in Earth's atmosphere, sound travels at different speeds. The speed of sound $s(x)$ (in meters per second) can be modeled by

$$s(x) = \begin{cases} -4x + 341 & \text{if } 0 \le x < 11.5 \\ 295 & \text{if } 11.5 \le x < 22 \end{cases}$$

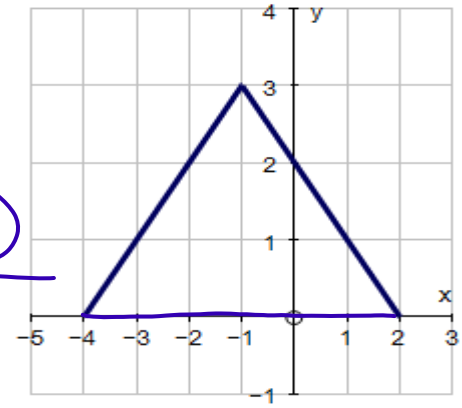
where x is measured in ~~kilometers~~ seconds. What is the average speed of sound over the interval [0, 22]?

$$\frac{\int_0^{11.5} (-4x+341)\,dx + \int_{11.5}^{22} 295\,dx}{22-0} = 307.023 \text{ m/s}$$

9. Find the average value of the function on the interval *without integrating*, by appealing to the geometry of the region between the graph and the *x*-axis.

a) $f(x)=\begin{cases} x+4 & -4\le x\le -1 \\ -x+2 & -1<x\le 2 \end{cases}$ on [–4, 2]

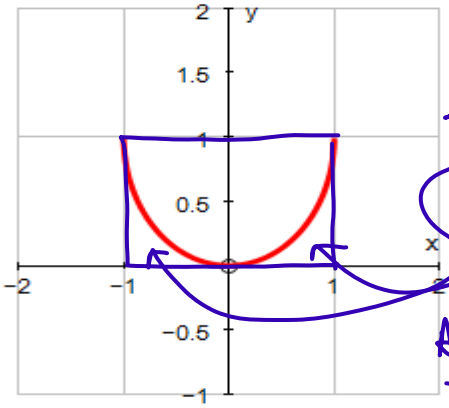
b) $f(x)=1-\sqrt{1-x^2}$ on [–1, 1]



$\frac{1}{2}(6)(3)$

$\frac{6}{}$

$=3/2$



$2-\frac{1}{2}\pi(1)^2$

$2-\frac{\pi}{2}$

Av value $=\frac{2-\frac{\pi}{2}}{2}$ or $1-\frac{\pi}{4}$

10. Set up an integral to find the average value of the functions in the last question, then use your calculator to evaluate.

a) $\frac{\int_{-4}^{-1}(x+4)\,dx+\int_{-1}^{2}(-x+2)\,dx}{2--4}$

b) $\frac{\int_{-1}^{1}(1-\sqrt{1-x^2})\,dx}{1--1}$

11. [Calculator] Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function *F* defined by

$$F(t)=82+4\sin\left(\frac{t}{2}\right) \text{ for } 0\le t\le 30,$$

where *F* (*t*) is measured in cars per minute and *t* is measured in minutes.

a) Is traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.

$F'(7)=-1.873$ cars/min/min Traffic Flow is decreasing since $F'(7)<0$

b) What is the average value of the traffic flow over the time interval $10\le t\le 15$? Indicate units of measure.

$\frac{\int_{10}^{15}F(t)\,dt}{15-10}=81.899$ cars/min

c) What is the average rate of change of the traffic flow over the time interval $10\le t\le 15$? Indicate units of measure

$\frac{F(15)-F(10)}{15-10}=1.518$ cars/min/min