

AP Calculus  
5.4 Worksheet Day 1

*All work must be shown in this course for full credit. Unsupported answers may receive NO credit.*

**For questions 1 – 10, use the Fundamental Theorem of Calculus (Evaluation Part) to evaluate each definite integral. Use your memory of derivative rules and/or the chart from your notes. You should start making a list of all the rules on ONE page!**

1.  $\int_1^4 \left( x^3 + \frac{5}{\sqrt{x}} \right) dx$

2.  $\int_3^5 \frac{dx}{x}$

3.  $\int_{\frac{1}{2}}^{\frac{\sqrt{5}}{2}} \frac{1}{\sqrt{1-x^2}} dx$

4.  $\int_{-1}^{\sqrt{5}} \frac{1}{1+x^2} dx$

5.  $\int_0^2 5^x dx$

6.  $\int_{-5}^{12} 7x dx$

7.  $\int_{-2}^5 6 dx$

8.  $\int_{\frac{1}{2}}^{\pi} 5 \sin(x) dx$

9.  $\int_0^{\frac{7}{4}} \sec^2(x) dx$

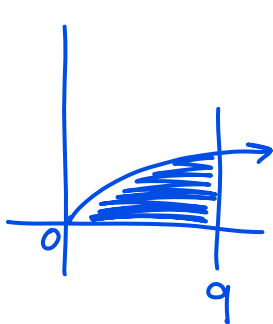
10.  $\int_{-1}^3 e^x dx$

*If you would like more practice with the FTOC (Evaluation part)? ... page 303 #27 – 40 (ask to borrow a book)*

For questions 11 and 12, setup and evaluate an expression involving definite integrals in order to find the total AREA of the region between the curve and the x-axis. [No Calculator!]

11.  ~~$y = 3x^2 - 3$  on the interval  $-2 \leq x \leq 2$~~

12.  $y = \sqrt{x}$  on the interval  $0 \leq x \leq 9$



$$\int_0^9 \sqrt{x} dx = \int_0^9 x^{\frac{1}{2}} dx = \frac{2}{3} x^{\frac{3}{2}} \Big|_0^9 = \frac{2}{3} 9^{\frac{3}{2}} = \frac{2}{3} (3^2)^{\frac{3}{2}} = \frac{2}{3} (3)^3 = 2(3)^2 = 18$$

For questions 13 – 16, find the average value of the function on the specified interval without a calculator.

13.  ~~$g(x) = 9 - 3x^2$  on the interval  $[0, 4]$~~

14.  $h(x) = \csc(x) \cot(x)$  on the interval  $[\frac{\pi}{4}, \frac{\pi}{2}]$

$$\frac{\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \csc x \cot x dx}{\frac{\pi}{2} - \frac{\pi}{4}} = \frac{-\csc x \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}}{\frac{\pi}{4}} = \frac{-\csc(\frac{\pi}{2}) + \csc(\frac{\pi}{4})}{\frac{\pi}{4}} = \frac{-1 + \sqrt{2}}{\frac{\pi}{4}} = (-1 + \sqrt{2}) \cdot \frac{4}{\pi}$$

15.  $y = \begin{cases} 5x & \text{if } 0 \leq x \leq 2 \\ 12 - x & \text{if } 2 < x \leq 12 \end{cases}$

16.  $f(x) = \sec^2 x$  on the interval  $[0, \frac{\pi}{4}]$

$$\frac{\int_0^{\frac{\pi}{4}} \sec^2 x dx}{\frac{\pi}{4} - 0} = \frac{\tan x \Big|_0^{\frac{\pi}{4}}}{\frac{\pi}{4}} = \frac{\tan \frac{\pi}{4} - \tan 0}{\frac{\pi}{4}} = \frac{1}{\frac{\pi}{4}} \text{ or } \frac{4}{\pi}$$

17. Including start-up costs, it costs a printer \$50 to print 24 copies of a newsletter, after which the marginal cost (in dollars per copy) at  $x$  copies is given by  $C'(x) = \frac{2}{\sqrt{x}}$ . Find the total cost of printing 2500 newsletters.

$$C(2500) = 50 + \int_{25}^{2500} \frac{2}{\sqrt{x}} dx$$

$$50 + \int_{25}^{2500} 2x^{-\frac{1}{2}} dx$$

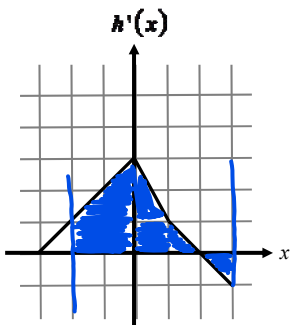
$$50 + \left[ 4x^{\frac{1}{2}} \right]_{25}^{2500} = 50 + 4\sqrt{2500} - 4\sqrt{25} = 50 + 4(50) - 4(5) = 230$$

18. If you know  $\int_{-7}^9 f'(x) dx = 15$ , and you know  $f(-7) = 4$ , what does  $f(9) = ?$

$$15 = \int_{-7}^9 f'(x) dx = f(9) - f(-7) = 15$$

$$f(9) - 4 = 15 \quad \text{so } f(9) = 19$$

19. The graph of  $h'(x)$  is given below. If  $h(-2) = 6$ , what does  $h(3) = ?$



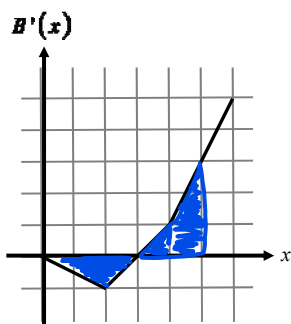
$$\int_{-2}^3 h'(x) dx = h(3) - h(-2)$$

$$4 + 2$$

$$6 = h(3) - 6$$

$$h(3) = 12$$

20. The graph of  $B'(x)$  is given below. If you know that  $B(0) = 5$ , what does  $B(5) = ?$



$$\int_0^5 B'(x) dx = B(5) - B(0)$$

$$-1 + 1 + 2 = B(5) - 5$$

$$1 = B(5) - 5$$

$$B(5) = 6$$