AP Calculus
5.4 Worksheet Day 1

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

For questions 1 – 10, use the Fundamental Theorem of Calculus (Evaluation Part) to evaluate each definite integral. Use your memory of derivative rules and/or the chart from your notes. You should start making a list of all the rules on ONE page!

1. $\int_1^4 \left(x^3 + \frac{5}{\sqrt{x}}\right)dx$

$5x^{-\frac{1}{2}}$

$\frac{x^4}{4} + 10x^{\frac{1}{2}} \Big|_1^4$

$\left[\frac{4^4}{4} + 10(4)^{\frac{1}{2}}\right] - \left[\frac{1^4}{4} + 10(1)^{\frac{1}{2}}\right]$

$64 + 20 - \frac{1}{4} - 10$

$74 - \frac{1}{4} = \boxed{73\frac{3}{4}}$

2. $\int_3^5 \frac{dx}{x} \rightarrow \int_3^5 \frac{1}{x}\,dx$

$= \ln|x| \Big|_3^5 = \ln 5 - \ln 3$

$= \ln\left(\frac{5}{3}\right)$

3. $\int_{1/2}^{\sqrt{3}/2} \frac{1}{\sqrt{1-x^2}}\,dx$

$= \sin^{-1} x \Big|_{1/2}^{\sqrt{3}/2}$

$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$

4. $\int_{-1}^{\sqrt{3}} \frac{1}{1+x^2}\,dx$

$\tan^{-1} x \Big|_{-1}^{\sqrt{3}} = \tan^{-1}\sqrt{3} - \tan^{-1}(-1)$

$= \frac{\pi}{3} - -\frac{\pi}{4} = \frac{7\pi}{12}$

5. $\int_0^2 5^x\,dx$

$\frac{5^x}{\ln 5} \Big|_0^2 = \frac{1}{\ln 5}(25 - 1)$

$(5^2 - 5^0)$

$= \frac{24}{\ln 5}$

6. $\int_{-5}^{12} 7x\,dx$

$\frac{7x^2}{2} \Big|_{-5}^{12}$

$\frac{7}{2}(12^2 - (-5)^2) = \frac{7}{2}(144 - 25) = \frac{7}{2}(119)$

7. $\int_{-2}^{5} 6\,dx = 6x \Big|_{-2}^{5}$

$6(5) - 6(-2) = 30 + 12 = 42$

8. $\int_{\pi/2}^{\pi} 5\sin(x)\,dx$

$-5\cos x \Big|_{\pi/2}^{\pi}$

$-5\cos\pi + 5\cos\frac{\pi}{2}$

5

9. $\int_0^{\pi/4} \sec^2(x)\,dx$

$= \tan x \Big|_0^{\pi/4}$

$= \tan\frac{\pi}{4} - \tan 0 = 1$

10. $\int_{-1}^{3} e^x\,dx$

$e^x \Big|_{-1}^{3} = e^3 - \frac{1}{e}$

$= \frac{e^4 - 1}{e}$

If you would like more practice with the FTOC (Evaluation part)? … page 303 #27 – 40 (ask to borrow a book)

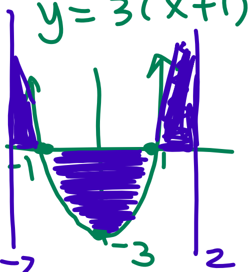
For questions 11 and 12, setup and evaluate an expression involving definite integrals in order to find the total AREA of the region between the curve and the *x*-axis. [No Calculator!]

11. $y = 3x^2 - 3$ on the interval $-2 \le x \le 2$

12. $y = \sqrt{x}$ on the interval $0 \le x \le 9$

$y = 3(x^2 - 1)$

$y = 3(x+1)(x-1)$



$2\int_1^2 (3x^2 - 3)\,dx - \int_{-1}^{1} (3x^2 - 3)\,dx$

$2\left[x^3 - 3x\right]_1^2 - \left[x^3 - 3x\right]_{-1}^{1}$

$2(2^3 - 3(2)) - 2(1^3 - 3(1)) - \{(1^3 - 3(1)) - ((-1)^3 - 3(-1))\}$

$= 2(8-6) - 2(1-3) - (1-3) + (-1+3) = 4+4+2+2 = 12$

For questions 13 – 16, find the average value of the function on the specified interval without a calculator.

13. $g(x) = 9 - 3x^2$ on the interval [0, 4]

14. $h(x) = \csc(x)\cot(x)$ on the interval $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

$\frac{\int_0^4 (9 - 3x^2)\,dx}{4-0} = \frac{\left[9x - x^3\right]_0^4}{4} = \frac{9(4) - 4^3}{4} = 9 - 4^2 = -7$

15. $y = \begin{cases} 5x & \text{if } 0 \le x \le 2 \\ 12 - x & \text{if } 2 < x \le 12 \end{cases}$

16. $f(x) = \sec^2 x$ on the interval $\left[0, \frac{\pi}{4}\right]$

$\frac{\int_0^2 5x\,dx + \int_2^{12} (12 - x)\,dx}{12 - 0} = \frac{1}{12}\left[\frac{5x^2}{2}\Big|_0^2 + \left[12x - \frac{x^2}{2}\right]_2^{12}\right]$

$= \frac{1}{12}\left[\frac{5(2)^2}{2} + 12(12) - \frac{12^2}{2} - (12)(2) + \frac{2^2}{2}\right]$

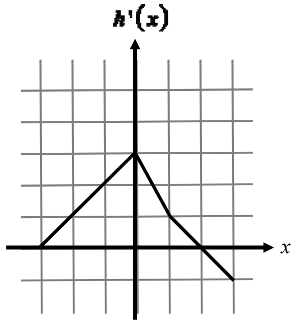
17. Including start-up costs, it costs a printer \$50 to print 24 copies of a newsletter, after which the marginal cost (in dollars per copy) at x copies is given by $C'(x) = \frac{2}{\sqrt{x}}$. Find the total cost of printing 2500 newsletters.

$= \frac{1}{12}\left[10 + 144 - 72 - 24 + 2\right]$

$= \frac{1}{12}\left[60\right] = 5$

18. If you know $\int_{-7}^{9} f'(x)\,dx = 15$, and you know $f(-7) = 4$, what does $f(9) = ?$

19. The graph of $h'(x)$ is given below. If $h(-2) = 6$, what does $h(3) = ?$



20. The graph of $B'(x)$ is given below. If you know that $B(0) = 5$, what does $B(5) = ?$