You are given a table containing some values of differentiable functions $f(x)$, $g(x)$ and their derivatives. Use the table data and the rules of differentiation to solve each problem.

6)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	1	2	6	−1
2	3	$\frac{3}{2}$	5	−1
3	4	1	4	−1
4	5	1	3	−1
5	6	$-\frac{1}{2}$	2	−1
6	4	−2	1	−1

Part 1) Given $h_1(x) = f(x) + g(x)$, find $h_1'(6)$ Sum

Part 2) Given $h_2(x) = f(x) - g(x)$, find $h_2'(3)$ Difference

Part 3) Given $h_3(x) = f(x) \cdot g(x)$, find $h_3'(1)$ Product

Part 4) Given $h_4(x) = \frac{f(x)}{g(x)}$, find $h_4'(5)$ Quotient

1

$h_1'(x) = f'(x) + g'(x)$

$h_1'(6) = f'(6) + g'(6) = -2 + (-1) = -3$

2 $h_2'(x) = f'(x) - g'(x)$

$h_2'(3) = f'(3) - g'(3)$

$= 1 - (-1)$

$= 2$

3 $h_3'(x) = f'(x) \cdot g(x) + f(x)\, g'(x)$

$h_3'(1) = f'(1)\, g(1) + f(1)\, g'(1)$

$= (2)(6) + (1)(-1) = 12 + (-1) = 11$

4 $h_4'(x) = \frac{g(x) \cdot f'(x) - f(x)\, g'(x)}{[g(x)]^2}$

$h_4'(5) = \frac{g(5)\, f'(5) - f(5)\, g'(5)}{[g(5)]^2}$

$= \frac{(2)(-\frac{1}{2}) - (6)(-1)}{(2)^2} = \frac{-1+6}{4} = \frac{5}{4}$

For each problem, find the points (x, y) where the tangent line to the function is horizontal

7) $y = \frac{-x^2}{5x+4}$

$\frac{dy}{dx} = \frac{(5x+4)(-2x) - (-x^2)(5)}{(5x+4)^2} = 0$

Fractions = 0 if numerator = 0 (and denom ≠ 0)

$-10x^2 - 8x + 5x^2 = 0$

$-5x^2 - 8x = 0$

$-x(5x+8) = 0$

$\boxed{x = 0, -\frac{8}{5}}$

$\boxed{y(0) = 0}$

$y(-\frac{8}{5}) = \frac{-(-\frac{8}{5})^2}{5(-\frac{8}{5})+4}$

$= \frac{-\frac{64}{25}}{-8+4} = \frac{-\frac{64}{25}}{-4} = \frac{16}{25}$

$\boxed{(0,0)\quad (-\frac{8}{5}, \frac{16}{25})}$

Evaluate each limit.

8) $\lim_{h \to 0} \frac{\left(\frac{1}{3}+h\right)^4 - \left(\frac{1}{3}\right)^4}{h}$ for x

$f(x) = x^4$

$f'(x) = 4x^3$

$f'(\frac{1}{3}) = 4(\frac{1}{3})^3 = \frac{4}{27}$

9) $\lim_{h \to 0} \frac{\sqrt{4+h} - 2}{h}$ x

$f(x) = \sqrt{x} \rightarrow$ call it $x^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} \rightarrow \frac{1}{2\sqrt{x}}$

$f'(4) = \frac{1}{2\sqrt{4}} \rightarrow \frac{1}{2 \cdot 2} = \frac{1}{4}$