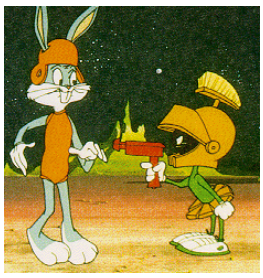
AP Calculus
3.4 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. What is the relationship between position, velocity, and acceleration?

$v(t) = s'(t)$ $a(t) = v'(t) = s''(t)$

2. Once again trying to blow up earth because it interferes with his view of Venus, Marvin the Martian lands on the moon. Bugs Bunny, as always, interferes with his plan. Chasing Bugs, Marvin fires a warning shot straight up into the air with his Acme Disintegration Pistol. The height (in feet) after t seconds of the shot is given by



$$s(t) = -2.66t^2 + 135t + 3.$$

a) Find the velocity and acceleration as functions of time. (What is the meaning of the acceleration function?)

$v(t) = s'(t) = -5.32t + 135$ (Gravity on the moon)

$a(t) = s''(t) = -5.32$

b) What is the position of the shot when the velocity is 0?

$0 = -5.32t + 135 \Rightarrow t = 25.376$

$s(25.376) = 1715.876$ ft

3. Fill in the blanks.

a) When the velocity is positive, the object is moving in a positive direction.

b) An object is slowing down when the velocity and acceleration have different signs.

c) An object is stopped when velocity is zero.

d) Speed is always positive because it is the absolute value of velocity.

4. A bug begins to crawl up a vertical wire at time $t = 0$. The velocity, v, of the bug at time t, $0 \le t \le 8$ is given by the function whose graph is shown below.

a) At what value of t does the bug change direction? Justify your response.

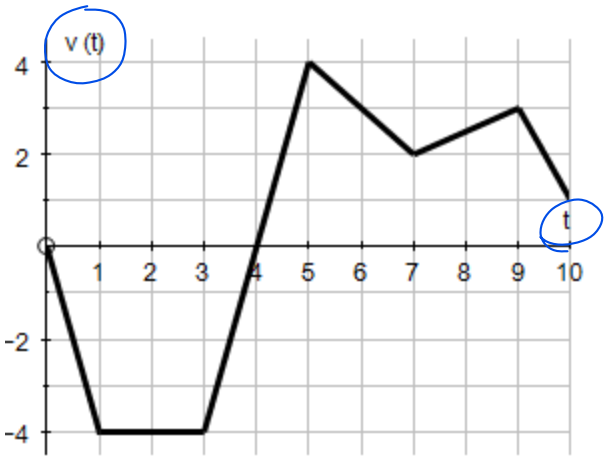
$t = 6$ $v(t)$ changes sign

b) During which time intervals in the bug slowing down? Justify your response.

$(4,6)$ $(7,8)$

$v(t)$ and $a(t)$ are different signs

5. The figure graphed below shows the velocity of a particle moving along a coordinate line. Justify each response.



a) When is the particle moving right?

$(4,10)$, $v(t) > 0$

b) When is the particle moving left?

$(0,4)$, $v(t) < 0$

c) When is the particle stopped?

$t = 0, 4$, $v(t) = 0$

d) When is the particle speeding up?

$(0,1)$ $(4,5)$ $(7,9)$, $v(t)$ and $a(t)$ have same sign

e) When does the particle change directions?

$t = 4$, $v(t)$ change sign

f) When is the particle slowing down?

$(3,4)$ $(5,7)$ $(9,10)$, $v(t)$ and $a(t)$ have opposite signs

g) ~~What~~ When is the particle moving at its greatest speed?

$(1,3)$ and $t = 5$ (speed $= 4$)

h) When is the particle's acceleration positive?

$(3,5)$ $(7,9)$, slope of $v(t) > 0$

i) When is the particle's acceleration negative?

$(0,1)$ $(5,7)$ $(9,10)$, slope of $v(t) < 0$

6. Fill in the blanks with correct mathematical notation. $s(t)$ is position function

a) If you want the average velocity of a particle on the interval [2, 5], you must find $\dfrac{s(5) - s(2)}{5 - 2}$.

b) If you want the velocity of a particle at $t = 4$, you must find $s'(4)$.

7. Velocity is the rate of change of position. If the position of a particle on the x – axis at time t is given by $-5t^2$, then what is the average velocity of the particle for $0 \le t \le 3$?

$s(t) = -5t^2$

$s(3) = -5(3)^2 = -5 \cdot 9 = -45$

$s(0) = -5(0)^2 = -5 \cdot 0 = 0$

$\dfrac{s(3) - s(0)}{3 - 0} = \dfrac{-45 - 0}{3} = -15$

8. A particle moves along the x – axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

$x'(t) = 2t - 6$

$2t - 6 = 0$

$t = 3$

9. Fill in the blanks with correct mathematical notation.

a) If you want the average acceleration of a particle on the interval [1, 3], you must find $\dfrac{v(3) - v(1)}{3 - 1}$.

b) If you want the acceleration of a particle at $t = 8$, you must find $v'(8)$.

10. Rocket A has a positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds as shown in the table below.

t (sec)	0	10	20	30	40	50	60	70	80
$v(t)$ (ft/sec)	5	14	22	29	35	40	44	47	49

a) Find the average acceleration of Rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

$$\frac{v(80)-v(0)}{80-0} = \frac{49-5}{80} \text{ ft/s}^2$$

b) Using the data in the table, find an estimate for $v'(35)$. Indicate units of measure.

$$v'(35) \approx \frac{v(40)-v(30)}{40-30} = \frac{35-29}{10} \text{ ft/s}^2$$

$-9+1=-8$

$27-36+1$

11. A particle moves along the x–axis so that its position at any time $t \ge 0$ is given by the function $x(t) = t^3 - 12t + 1$, where x is measured in feet and t is measured in seconds. Justify each response and indicate units of measure when appropriate.

a) Find the displacement during the first 3 seconds.

$x(3) - x(0)$

$x(3) = -8 \quad x(0) = 1$

$-8-1 = -9$ ft

b) Find the average velocity during the first 3 seconds.

$$\frac{x(3)-x(0)}{3-0} = \frac{-9}{3} = -3 \text{ ft/s}$$

c) Find the instantaneous velocity at $t = 3$ seconds.

$x'(t) = 3t^2 - 12$

$x'(3) = 3(3)^2 - 12 = 15$ ft/s

d) Find the acceleration when $t = 3$ seconds.

$x''(t) = 6t$

$x''(3) = 18$ ft/s^2

e) When is the particle moving left?

$3t^2 - 12 = 0$

$t^2 = 4$ so $t = 2$

− | + (2)

$(0,2)$ $v(t) < 0$

f) At what value(s) t does the particle change direction?

$t = 2$ $v(t)$ changes sign

g) When is the particle speeding up?

$a(t) = 6t = 0 \quad t = 0$

− | + (0)

that's because one factor starts

$(2, \infty)$

b/c $v(t)$ and $a(t)$ have same sign

13. [Calculator] The cost involved in maintaining annual inventory for a certain manufacturer is given by $C(x) = \frac{1{,}008{,}000}{x} + 6.3x$, where x is the number of items stored. Find the marginal cost of storing the 351st item.

$C(x) = 1{,}008{,}000x^{-1} + 6.3x$

$C'(x) = -1{,}008{,}000x^{-2} + 6.3$

$C'(350) = -1.93$

saving $

14. [Calculator] Suppose that the dollar cost of producing x washing machines is $c(x) = 2000 + 100x - 0.1x^2$.

a) Find the marginal cost when 100 washing machines are produced.

$c'(x) = 100 - 2x$ $\quad c'(100) = 100 - 2(100) = 100 - 20 = \$80$ / machine

b) Show that the marginal cost when 100 washing machines are produced (your answer from part *b*) is approximately the cost of producing one more washing machine after the first 100 have been made, by calculating the latter cost directly.

$C(101) - C(100) = 11{,}079.9 - 11{,}000 = \$79.90$ for the 101st machine

15. [Calculator] Suppose the weekly revenue ($) from selling x custom-made office desks is $r(x) = 2000\left(1 - \frac{1}{x+1}\right)$.

Find the marginal revenue when a 6th desk is created.

$r(x) = 2000 + \frac{-2000}{x+1}$

$r'(x) = \frac{(x+1)\,0 - (-2000)(1)}{(x+1)^2}$, $r'(5) = \frac{2{,}000}{36}$

16. a) Write the area A of a circle as a function of the circumference C.

$A = \pi r^2$ $\quad C = 2\pi r \rightarrow r = \frac{C}{2\pi} \rightarrow A = \pi\left(\frac{C}{2\pi}\right)^2 = \frac{\pi C^2}{4\pi^2} = \frac{C^2}{4\pi}$

b) Evaluate the rate of change of A at $C = 4\pi$.

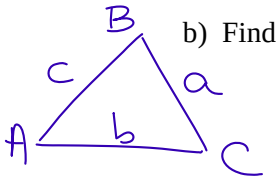
$\frac{dA}{dC} = \frac{2C}{4\pi}$ or $\frac{C}{2\pi}$

c) If C is measured in miles and A is measured in square miles, what units are used for $\frac{dA}{dC}$? $\frac{mi^2}{mi} = miles$

17. a) Write the area A of an equilateral triangle as a function of the side length s.

$A = \frac{1}{2}bh$, $A = \frac{1}{2}ab\sin C$

b) Find $\left.\frac{dA}{ds}\right|_{s=12}$



$A = \frac{1}{2}ss\sin 60^\circ = \frac{1}{2}s^2\frac{\sqrt{3}}{2} = \frac{s^2\sqrt{3}}{4}$

$\frac{dA}{ds} = \frac{2s\sqrt{3}}{4}$ $\quad \left.\frac{dA}{ds}\right|_{s=12} = \frac{2(12)\sqrt{3}}{4} = 6\sqrt{3}$

18. The number of gallons of water in a tank m minutes after the tank has started to drain is given by the equation $G(m) = 300(20 - t^2)$. $\rightarrow 6000 - 300t^2$

a) How fast is the water draining at the end of 5 minutes?

$G'(m) = 300(-2t) = -600t$

$G'(5) = -3000$ gallons / min

b) What is the average rate at which the water drains out of the tank during the first 5 minutes?

-1500 gallons / min

$\frac{G(5) - G(0)}{5 - 0} = \frac{[300(20-25)] - [300(20-0)]}{5}$

$= \frac{-1500 - 6000}{5} = \frac{-7500}{5}$