AP Calculus
3.9 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Suppose $\frac{d}{dx}\left[10 = e^{xy} + x^2 + y^2\right]$, find $\frac{dy}{dx}$.

$$0 = e^{xy}\left(1 \cdot y + x\frac{dy}{dx}\right) + 2x + 2y\frac{dy}{dx}$$

$$0 = ye^{xy} + xe^{xy}\frac{dy}{dx} + 2x + 2y\frac{dy}{dx}$$

$$-ye^{xy} - 2x = \frac{dy}{dx}\left(xe^{xy} + 2y\right)$$

$$\boxed{\frac{dy}{dx} = \frac{-ye^{xy} - 2x}{xe^{xy} + 2y}}$$

2. Find $g'(t)$ if $g(t) = t^e\left(e^{-t}\right)$

$$g'(t) = t^e e^{-t}(-1) + et^{e-1} \cdot e^{-t}$$

$$= e^{-t}\left(-t^e + et^{e-1}\right)$$

3. Find $g'(t)$ if $g(t) = \ln(\ln t)$.

$$g'(t) = \frac{1}{\ln t} \cdot \frac{1}{t} \quad \text{or} \quad \frac{1}{t\ln t}$$

4. Use properties of logarithms to rewrite $h(x)$ and then find $h'(x)$ if $h(x) = \ln\left(\frac{1+e^x}{1-e^x}\right)$. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$$h(x) = \ln(1+e^x) - \ln(1-e^x)$$

$$h'(x) = \frac{1}{1+e^x}(e^x) - \frac{1}{1-e^x}(-e^x) \quad \text{OR} \quad \frac{e^x}{1+e^x} + \frac{e^x}{1-e^x}$$

5. Find the first derivative for $y = x^{\ln x}$ (use logarithmic differentiation).

$$\ln y = \ln x^{\ln x}$$

$$\frac{d}{dx}\left[\ln y = \ln x \ln x\right]$$

$$y\left[\frac{1}{y}\frac{dy}{dx} = \ln x \cdot \frac{1}{x} + \frac{1}{x}\ln x\right]y$$

$$\frac{dy}{dx} = \frac{2\ln x}{x}y$$

$$\boxed{\frac{dy}{dx} = \frac{2\ln x}{x} \cdot x^{\ln x}}$$

6. Find y' $\overset{\frac{dy}{dx}}{}$ if $y=\frac{x^3}{3^x}$ first using the quotient rule, then using logarithmic differentiation.

$$\frac{dy}{dx}=\frac{\cancel{3^x}\cdot 3x^2 - x^3\cdot\cancel{3^x}\cdot\ln 3}{(3^x)^{\cancel{2}}}$$

$$=\frac{3x^2-x^3\ln 3}{3^x}$$

$$\ln y=\ln\left(\frac{x^3}{3^x}\right)$$

$$\frac{d}{dx}\left[\ln y=\ln(x^3)-\ln(3^x)\right]$$

$$\frac{1}{y}\cdot\frac{dy}{dx}=\frac{1}{x^3}(3x^2)-\frac{1}{3^x}(3^x\ln 3)$$

$$y\left[\frac{1}{y}\frac{dy}{dx}=\frac{3}{x}-\ln 3\right]y$$

$$\frac{dy}{dx}=\left(\frac{3}{x}-\ln 3\right)\left(\frac{x^3}{3^x}\right)$$

7. Solve the following without using a calculator: If $f(x)=(x^2+1)^{(2-3x)}$, then $f'(1)=$

$\ln y=\ln(x^2+1)^{(2-3x)}$

A $-\frac{1}{2}\ln(8e)$ B $-\ln(8e)$ C $-\frac{3}{2}\ln(2)$ D $-\frac{1}{2}$ E $\frac{1}{8}$

$y(1)=(1^2+1)^{(2-3(1))}$

$y(1)=2^{-1}=\frac{1}{2}$

$\ln y=(2-3x)\ln(x^2+1)$

$$\left[\frac{1}{y}\frac{dy}{dx}=(2-3x)\frac{1}{x^2+1}(2x)+(-3)\ln(x^2+1)\right]y$$

$$\left.\frac{dy}{dx}\right|_{(1,\frac{1}{2})}=\left[(2-3(1))\frac{1}{1^2+1}(2\cdot 1)+(-3)\ln(1^2+1)\right]\frac{1}{2}\rightarrow\left[-1-3\ln 2\right]\frac{1}{2}$$

$(1+3\ln 2)\left(-\frac{1}{2}\right)$

$(1+\ln 8)\left(\frac{-1}{2}\right)$

$(\ln e+\ln 8)\left(\frac{-1}{2}\right)$

$(\ln 8e)\left(-\frac{1}{2}\right)$

8. If $y=\tan u$, $u=v-\frac{1}{v}$, and $v=\ln x$, what is the value of $\frac{dy}{dx}$ at $x=e$?

$u=v-1v^{-1}$

A 0 B $\frac{1}{e}$ C 1 D $\frac{2}{e}$ E $\sec^2(e)$

$$\frac{dy}{du}=\sec^2 u$$

$$\frac{dy}{du}\ \frac{du}{dv}\ \frac{dv}{dx}$$

$v=\ln e=1$

$u=1-\frac{1}{1}=0$

$$\frac{du}{dv}=1+1v^{-2}$$

$$\sec^2 u\ (1+v^{-2})\left(\frac{1}{x}\right)$$

$$\sec^2(0)(1+1^{-2})\left(\frac{1}{e}\right)$$

$$\frac{dv}{dx}=\frac{1}{x}$$

$$(1)(2)\left(\frac{1}{e}\right)$$