AP Calculus
3.9 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Suppose $10 = e^{xy} + x^2 + y^2$, find $\frac{dy}{dx}$.

$$0 = e^{xy}\cdot\left(x\frac{dy}{dx} + 1y\right) + 2x + 2y\frac{dy}{dx}$$

$$0 = e^{xy}x\frac{dy}{dx} + e^{xy}y + 2x + 2y\frac{dy}{dx}$$

$$-e^{xy}y - 2x = \frac{dy}{dx}\left(e^{xy}x + 2y\right)$$

$$\boxed{\frac{dy}{dx} = \frac{-e^{xy}y - 2x}{e^{xy}x + 2y}}$$

2. Find $g'(t)$ if $g(t) = t^e\left(e^{-t}\right)$

$$g'(t) = \left(et^{e-1}\right)\left(e^{-t}\right) + \left(t^e\right)\left(e^{-t}\right)(-1)$$

$$= e^{-t}\left(et^{e-1} - t^e\right)$$

3. Find $g'(t)$ if $g(t) = \ln(\ln t)$.

$$\ln t \rightarrow \frac{1}{t}$$

$$\ln t \rightarrow \frac{1}{t}$$

$$\frac{1}{\ln t} \quad \frac{1}{t}$$

$$g'(t) = \frac{1}{\ln t}\,\frac{1}{t}$$

4. Use properties of logarithms to rewrite $h(x)$ and then find $h'(x)$ if $h(x) = \ln\left(\frac{1+e^x}{1-e^x}\right)$.

$$h(x) = \ln(1+e^x) - \ln(1-e^x)$$

$$h'(x) = \frac{1}{1+e^x}\cdot e^x - \frac{1}{1-e^x}\cdot -e^x$$

5. Find the first derivative for $y = x^{\ln x}$ (use logarithmic differentiation).

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(\ln x \ \ln x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\ln x + \ln x\frac{1}{x}$$

$$\frac{dy}{dx} = \frac{2\ln x}{x}\cdot y \quad \text{or} \quad \frac{2\ln x}{x}\cdot x^{\ln x}$$

6. Find y' if $y = \frac{x^3}{3^x}$ first using the quotient rule, then using logarithmic differentiation.

$$\frac{dy}{dx} = \frac{3^x \cdot 3x^2 - x^3 \cdot \ln 3 \cdot 3^x}{(3^x)^2}$$

$$= \frac{3x^2 - x^3 \ln 3}{3^x}$$

$$\ln y = \ln\left(\frac{x^3}{3^x}\right)$$

$$\ln y = \ln x^3 - \ln 3^x$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^3}(3x^2) - \frac{1}{3^x}(3^x)\ln 3$$

$$\frac{dy}{dx} = \left(\frac{3}{x} - \ln 3\right)\left(\frac{x^3}{3^x}\right)$$

7. Solve the following without using a calculator: If $f(x) = (x^2+1)^{(2-3x)}$, then $f'(1) =$ $f(1) = 2^{-1} = \frac{1}{2}$

$y = (x^2+1)^{(2-3x)}$

A $-\frac{1}{2}\ln(8e)$ B $-\ln(8e)$ C $-\frac{3}{2}\ln(2)$ D $-\frac{1}{2}$ E $\frac{1}{8}$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}\left((2-3x)\ \ln(x^2+1)\right)$$

$$\frac{1}{y}\frac{dy}{dx} = (-3)\ln(x^2+1) + (2-3x)\cdot\frac{1}{x^2+1}(2x)$$

$$\frac{1}{1/2}\frac{dy}{dx} = (-3)\ln(1^2+1) + (2-3)\cdot\frac{1}{1^2+1}(2\,1) \rightarrow 2\frac{dy}{dx} = -3\ln 2 + (-1)\left(\frac{2}{2}\right)$$

$$2\frac{dy}{dx} = -3\ln 2 - 1$$

$$\frac{dy}{dx} = \frac{-3\ln 2 - 1}{2}$$

8. If $y = \tan u$, $u = v - \frac{1}{v}$, and $v = \ln x$, what is the value of $\frac{dy}{dx}$ at $x = e$?

A 0 B $\frac{1}{e}$ C 1 D $\frac{2}{e}$ E $\sec^2(e)$