AP Calculus
5.4 Worksheet Day 1

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

For questions 1 – 10, use the Fundamental Theorem of Calculus (Evaluation Part) to evaluate each definite integral. Use your memory of derivative rules and/or the chart from your notes. You should start making a list of all the rules on ONE page!

1. $\int_1^4 \left(x^3 + \frac{5}{\sqrt{x}}\right) dx \rightarrow \int_1^4 (x^3 + 5x^{-\frac{1}{2}})\,dx$

$= \frac{x^4}{4} + \frac{5x^{\frac{1}{2}}}{1/2}$ → OR multiply by 2

$\left[\frac{4^4}{4} + 10(4)^{\frac{1}{2}}\right] - \left[\frac{1^4}{4} + 10(1)^{\frac{1}{2}}\right]$

$[64+20] - [\frac{1}{4} + 10]$

$84 - \frac{1}{4} - 10$

$74 - \frac{1}{4} = 73\frac{3}{4}$

2. $\int_3^5 \frac{dx}{x} \rightarrow \int_3^5 \frac{1}{x}\,dx$

$\rightarrow \ln|x|\Big|_3^5$

$\rightarrow \ln 5 - \ln 3 \rightarrow \ln\left(\frac{5}{3}\right)$

OK, but also

$\frac{4}{4}\,\frac{\pi}{3} + \frac{\pi}{4}\,\frac{3}{3}$ → $\frac{7\pi}{12}$

3. $\int_{1/2}^{\sqrt{3}/2} \frac{1}{\sqrt{1-x^2}}\,dx$

$\sin^{-1}x\Big|_{1/2}^{\sqrt{3}/2}$

$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$

4. $\int_{-1}^{\sqrt{3}} \frac{1}{1+x^2}\,dx$

$\tan^{-1}x\Big|_{-1}^{\sqrt{3}}$

$= \tan^{-1}(\sqrt{3}) - \tan^{-1}(-1) = \frac{\pi}{3} - \left(-\frac{\pi}{4}\right)$

5. $\int_0^2 5^x\,dx = \frac{5^x}{\ln 5}\Big|_0^2$

$\frac{1}{\ln 5}(5^2 - 5^0) = \frac{24}{\ln 5}$

6. $\int_{-5}^{12} 7x\,dx \quad \frac{7x^2}{2}\Big|_{-5}^{12}$

$\frac{7}{2}(12)^2 - \frac{7}{2}(-5)^2 = \frac{7(144)}{2} - \frac{7(25)}{2}$

7. $\int_{-2}^{5} 6\,dx = 6x\Big|_{-2}^{5}$

$6(6) - 6(-2)$

$30 + 12 = 42$

8. $\int_{\pi/2}^{\pi} 5\sin(x)\,dx = -5\cos x\Big|_{\pi/2}^{\pi}$

$-5\cos\pi + 5\cos\frac{\pi}{2}$

$-5(-1) + 5(0) = 5$

9. $\int_0^{\pi/4} \sec^2(x)\,dx$

$= \tan x\Big|_0^{\pi/4}$

$\tan\frac{\pi}{4} - \tan 0 = 1 - 0 = 1$

10. $\int_{-1}^{3} e^x\,dx \quad e^x\Big|_{-1}^{3}$

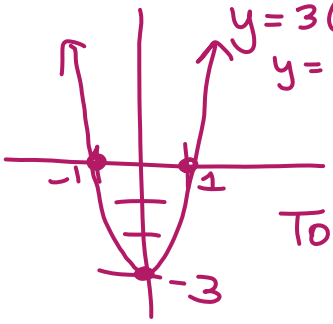
$= e^3 - e^{-1}$

$= \frac{e}{e}e^3 - \frac{1}{e} \Rightarrow \frac{e^4 - 1}{e}$

If you would like more practice with the FTOC (Evaluation part)? … page 303 #27 – 40 (ask to borrow a book)

For questions 11 and 12, setup and evaluate an expression involving definite integrals in order to find the total AREA of the region between the curve and the *x*-axis. [No Calculator!]

11. $y = 3x^2 - 3$ on the interval $-2 \le x \le 2$



Total Area $2\int_{-2}^{-1}(3x^2-3)\,dx - \int_{-1}^{1}(3x^2-3)\,dx$

12. $y = \sqrt{x}$ on the interval $0 \le x \le 9$

Total Area $\int_0^9 \sqrt{x}\,dx$ *entire graph in interval 0–9 is above x-axis

$\frac{\int_a^b f(x)\,dx}{b-a}$

For questions 13 – 16, find the average value of the function on the specified interval without a calculator.

13. $g(x) = 9 - 3x^2$ on the interval [0, 4]

$$\frac{\int_0^4 (9-3x^2)\,dx}{4-0} = \frac{1}{4}\left[9x - x^3\right]_0^4 = \frac{1}{4}(9\cdot 4 - 4^3) - \frac{1}{4}(9\cdot 0 - 0^3)$$
$$= \frac{1}{4}(36-64) = \frac{1}{4}(-28) = -7$$

14. $h(x) = \csc(x)\cot(x)$ on the interval $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

15. $y = \begin{cases} 5x & \text{if } 0 \le x \le 2 \\ 12 - x & \text{if } 2 < x \le 12 \end{cases}$

$$\frac{\int_0^2 5x\,dx + \int_2^{12}(12-x)\,dx}{12} \rightarrow \frac{\frac{5x^2}{2}\Big|_0^2 + \left[12x - \frac{x^2}{2}\right]_2^{12}}{12}$$

$$\frac{1}{12}\left[\frac{5(2)^2}{2} - \frac{5(0)^2}{2} + \left[12(12) - \frac{12^2}{2}\right] - \left[12(2) - \frac{2^2}{2}\right]\right] = \frac{1}{12}\left[10 + 144 - 72 - 24 + 2\right] = \frac{60}{12} = 5$$

16. $f(x) = \sec^2 x$ on the interval $\left[0, \frac{\pi}{4}\right]$

17. Including start-up costs, it costs a printer \$50 to print 24 copies of a newsletter, after which the marginal cost (in dollars per copy) at *x* copies is given by $C'(x) = \frac{2}{\sqrt{x}}$. Find the total cost of printing 2500 newsletters.

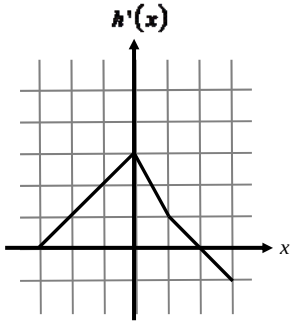
$$C(2500) = 50 + \int_{25}^{2500} C'(x)\,dx$$
$$= 50 + \int_{25}^{2500} 2x^{-\frac{1}{2}}\,dx$$
$$= 50 + 2\cdot 2x^{\frac{1}{2}}\Big|_{25}^{2500}$$
$$= 50 + 4(2500)^{\frac{1}{2}} - 4(25)^{\frac{1}{2}} = 50 + 4(50) - 4(5)$$

\$230

50 + 200 − 20

18. If you know $\int_{-7}^{9} f'(x)\,dx = 15$, and you know $f(-7) = 4$, what does $f(9) = ?$

19. The graph of $h'(x)$ is given below. If $h(-2) = 6$, what does $h(3) = ?$



20. The graph of $B'(x)$ is given below. If you know that $B(0) = 5$, what does $B(5) = ?$