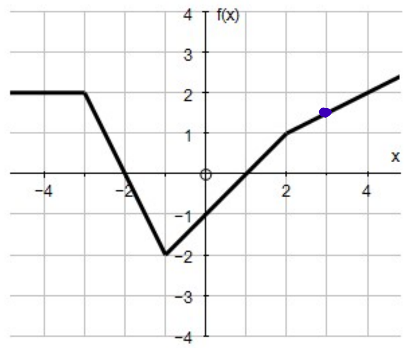
AP Calculus
5.4 Worksheet Day 2

$w'(x) = f(x)$

1. Let $w(x) = \int_1^x f(t)\,dt$. The graph of $f(x)$ is shown below.



a) Find $w(1)$

$w(1) = \int_1^1 f(t)\,dt = 0$

b) Find $w(3)$

$w(3) = \int_1^3 f(t)\,dt = 1.75$

c) Find $w(-2)$

$w(-2) = \int_1^{-2} f(t)\,dt = 3$

d) Find $w(-4)$

$w(-4) = \int_1^{-4} f(t)\,dt = 3 - 3 = 0$

e) What is $w'(x)$?

$f(x)$

f) Find $w'(2)$

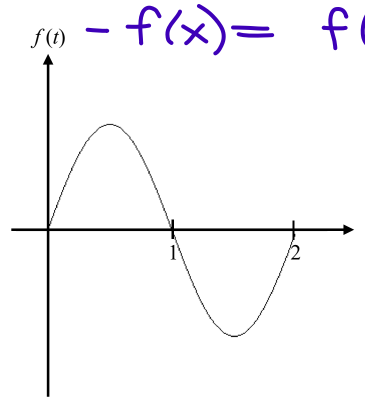
$w'(2) = f(2) = 1$

g) $w'(-1)$

$w'(-1) = f(-1) = -2$

2. Let $F(x) = \int_0^x f(t)\,dt$. The graph of $f(t)$ given below has <u>odd symmetry</u> and is periodic (with <u>period = 2</u>). If you know that $\int_0^1 f(t)\,dt = \frac{4}{3}$, complete the following table:

$-f(x) = f(-x)$



$\int_0^{-1} f(t)\,dt$

$\int_0^2 f(t)\,dt$

x	F(x)
–1	4/3
0	0
1	4/3
2	0
3	4/3

3. If a is a constant and $g(x) = \int_a^x w(t)\,dt$, what is $g'(x)$? <u>$w(x)$</u>

4. If a is a constant and $g(x) = \int_x^a w(t)\,dt$, what is $g'(x)$? <u>$-w(x)$</u>

5. Find $\frac{d}{dx}\left[\int_{-3}^{x} \sqrt{1+e^{5t}}\,dt\right]$.

$\sqrt{1+e^{5x}}$

6. If $y = \int_0^x \left(t^3 - t\right)^5 dt$, find y'.

$y' = (x^3 - x)^5$

7. $k(x)=\int_{-\pi}^{x}\frac{2-\sin u}{3+\cos u}\,du$. Find $k'(x)$.

$k'(x)=\frac{2-\sin x}{3+\cos x}$

8. Find $\frac{d}{dx}\left[\int_{x}^{7}\sqrt{2p^4+p+1}\,dp\right]$

$-\sqrt{2x^4+x+1}$

9. What is the linearization of $f(x)=\int_{\pi}^{x}\cos^3 t\,dt$ at $x=\pi$?

$f(\pi)=\int_{\pi}^{\pi}\cos^3 t\,dt=0$

$f'(x)=\cos^3(x)$

$f'(\pi)=\cos^3(\pi)$

$=(-1)^3$

$=-1$

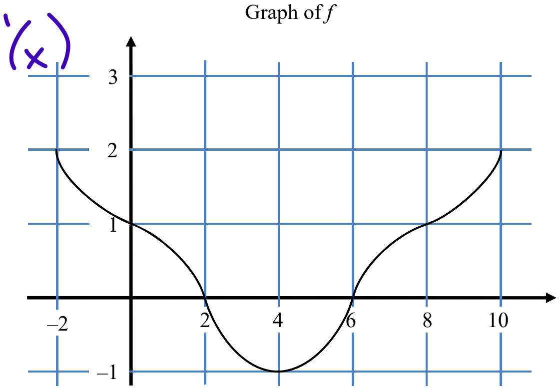
$\boxed{y-0=-1(x-\pi)}$

10. The graph of a differentiable function f on the interval [–2, 10] is shown in the figure below. The graph of f has a horizontal tangent line at $x = 4$.

Let $h(x)=9+\int_{4}^{x} f(t)\,dt$ for $-2<x<10$.

$h'(x)=f(x)\rightarrow h''(x)=f'(x)$

Graph of *f*



a) Find $h(4)$, $h'(4)$, and $h''(4)$

$h(4)=9+\int_{4}^{4}f(t)\,dt=9+0=9$

$h'(4)=f(4)=-1$

$h''(4)=f'(4)=0$

b) On what intervals is h increasing? Justify your answer.

Inc. $(-2,2)$ $(6,10)$ $h'(x)>0$ or $f>0$

c) On what intervals is h concave downward? Justify your answer.

cc down $(-2,4)$ f is decreasing

d) Find the Trapezoidal Sum to approximate $\int_{-2}^{10} f(x)\,dx$ using 6 subintervals of length = 2.

$2\cdot\frac{1}{2}\left(2+1(2)+0(2)+-1(2)+0(2)+1(2)+2\right)$

$1(2+2+0-2+0+2+2)=6$

11. If $q(x)$ and $p(x)$ are differential functions of x and $g(x) = \int_{q(x)}^{p(x)} w(t)\,dt$, what is $g'(x)$?

$g'(x) = w(p(x)) \cdot p'(x) - w(q(x))\; q'(x)$

12. Find $\frac{d}{dx}\left[\int_1^{\sin x} \sqrt{1+t^3}\,dt\right]$

$= \left(\sqrt{1+\sin^3 x}\right) \cos x$

13. Find $\frac{d}{dx}\left[\int_{x^2}^{x^3} \cos(2t)\,dt\right]$

$= \cos(2x^3)\, 3x^2 - \cos(2x^2)\, 2x$

14. If $y = \int_{3x^2}^{10} \ln(2+u^2)\,du$, find y'.

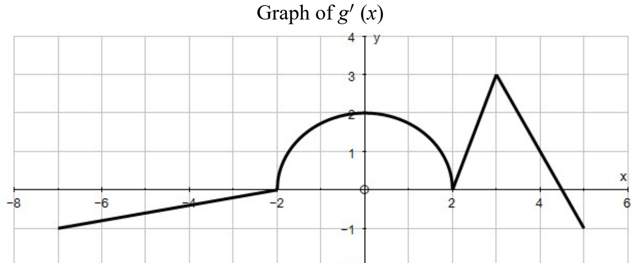
$y' = -\ln(2+9x^4)\; 6x$

15. Find $\frac{d}{dx}\left[\int_{\sin x}^{x^3} e^{t^2}\,dt\right]$

$= e^{x^6} \cdot 3x^2 - e^{\sin^2 x} \cdot \cos x$

16. The function g is define and differentiable on the closed interval [–7, 5] and satisfies $g(0) = 5$. The graph of $g'(x)$, the derivative of g, consists of a semicircle and three line segments as shown in the figure.

Graph of $g'(x)$



a) Write an expression for $g(x)$.

$g(x) = g(0) + \int_0^x g'(x)\,dx$

b) Use your expression to find $g(3)$ and $g(-2)$.

$g(3) = 5 + \int_0^3 g'(x)\,dx$

$= 5 + \pi + 1\,5 = 6.5 + \pi$

$g(-2) = 5 + \int_0^{-2} g'(x)\,dx$

$= 5 + (-\pi)$

$= 5 - \pi$

c) Find the x-coordinate of each point of inflection of the graph of $g(x)$ on the interval (–7, 5). Explain your reasoning.

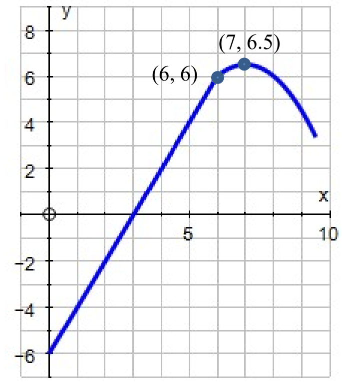
$g''(x)$ changes signs

$g''(x) = 0$ or undefined (Does not exist) $\rightarrow$ $x = -2, 0, 2, 3$

Point of inflection at $x = 0, 2, 3$ since $g''(x)$ change signs

17. Let $s(t) = \int_0^t f(x)\,dx$ be the position of a particle at time t (in seconds) as the particle moves along the x-axis. The graph of the differentiable function f is shown below. Use the graph to answer the following questions.

a) What is the particle's velocity at time $t = 4$? Justify your answer.



b) Is the acceleration of the particle at time $t = 4$ positive or negative? Justify your answer.

c) Is the particle speeding up or slowing down at time $t = 4$? Explain.

d) When does the particle pass through the origin? Explain.

e) Approximately when is the acceleration zero?

f) When is the particle moving toward the origin? Away from the origin?

g) On which side of the origin does the particle lie at time $t = 9$?