ABCALC Applications of Derivatives Review **Name:** ______________________

1. Find the absolute extrema of the function and where they occur.

a) $f(x) = 4x^2 - 4x - 3$ over $[-2, 2]$

$f'(x) = 8x - 4$

$0 = 8x - 4$

$x = \frac{1}{2}$

x	$f(x)$
-2	21
$\frac{1}{2}$	-4
2	5

abs max $= 21$ at $x = -2$

abs min $= -4$ at $x = 1/2$

b) (Calculator) $f(x) = (x^2 - 9x)^{\frac{1}{3}}$ over $[-4, 8]$

$f'(x) = \frac{1}{3}(x^2 - 9x)^{-\frac{2}{3}}(2x - 9)$

$= \dfrac{2x - 9}{3(x^2 - 9x)^{2/3}}$

$x^2 - 9x$

$x(x-9)$

$f'(x) = 0$ at $x = 9/2$

$f'(x)$ uno at $x = 0, 9$

x	$f(x)$
-4	3.733
0	0
$9/2$	-2.726
8	-2

abs max at

abs min at

2. Determine if MVT applies to the function. If it does, find the value of c guaranteed by the theorem. If it does not, explain why. cont on $[a,b]$ and diff on (a,b)

conditions ✓ ✓

a) $f(x) = 4x^2 + 5x$ over $[-2, 1]$

$f'(x) = 8x + 5$

$f(1) = 9$

$f(-2) = 6$

$\rightarrow \frac{6 - 9}{-2 - 1} \rightarrow \frac{-3}{-3} = 1$

$8x + 5 = 1$

$8x = -4$

$x = -\frac{1}{2}$ ✓

Cond ✓✓ b) (Calculator) $f(x) = \sin x$ over $[4, 5]$

$f'(x) = \cos x$

$f(4) = \sin 4$

$f(5) = \sin 5$

slope $= \frac{\sin(5) - \sin(4)}{5 - 4}$

$\frac{\sin(5) - \sin(4)}{5 - 4} = \cos x$

and solve for x

$x \approx 4.509$

3. For $f(x) = x^4 - 12x^2 - 13$, find the following.

a) Find the intervals that $f(x)$ is increasing and decreasing. Justify your response.

$f'(x) = 4x^3 - 24x$ C.P are

$f'(x) = 4x(x^2 - 6) = 0$ $x = 0, -\sqrt{6}, +\sqrt{6}$

$f'(x)$: $-$ on $(-\infty,-\sqrt{6})$, $+$ on $(-\sqrt{6},0)$, $-$ on $(0,\sqrt{6})$, $+$ on $(\sqrt{6},\infty)$

$f(x)$ inc $(-\sqrt{6}, 0)$ $(\sqrt{6}, \infty)$

$f'(x) > 0$

$f(x)$ dec $(-\infty, -\sqrt{6})$ $(0, \sqrt{6})$

$f'(x) < 0$

b) Find the x-values of all local minimum and maximum values. Justify each answer.

Local max is at $x = 0$

$f'(x)$ changes $+$ to $-$

Local mins at $x = -\sqrt{6}$ and $\sqrt{6}$

$f'(x)$ changes $-$ to $+$

c) Find all points of inflection. Justify your response.

$f''(x) = 12x^2 - 24$

$0 = 12(x^2 - 2)$

$f''(x)$: $+$ on $(-\infty,-\sqrt{2})$, $-$ on $(-\sqrt{2},\sqrt{2})$, $+$ on $(\sqrt{2},\infty)$

Po Inflection at $x = \pm\sqrt{2}$

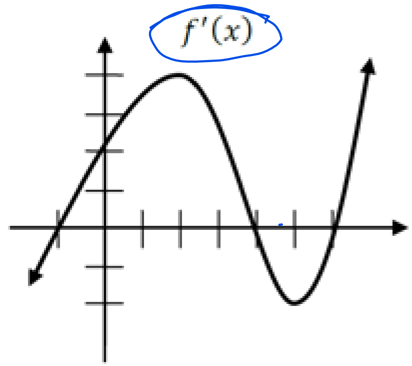
$f''(x)$ changes signs

ccup $(-\infty, -\sqrt{2})$ $(\sqrt{2}, \infty)$

d) Find the intervals that $f(x)$ is concave up and concave down. Justify your response.

ccdown $(-\sqrt{2}, \sqrt{2})$

4. Use the graph of $f'(x)$ to the right to answer the following. Justify each response.



Note: Graph of f'(x) not $f(x)$.

a) What is the slope of $f(x)$ at $x = 2$? $= 4$

slope of $f(x)$ IS the derivative $(f'(x))$

b) For which x values does $f(x)$ have a horizontal tangent line?

$x = -1, 4, 6$

c) Find the intervals where $f(x)$ is increasing. (when $f'(x) > 0$)

$(-1, 4)$ $(6, \infty)$

d) Find the x-values where $f(x)$ has a relative minimum/maximum. f' $-$ to $+$; f' $+$ to $-$

rel min at $x = -1$ and $x = 6$

rel max at $x = 4$

e) Is $f(x)$ increasing or decreasing at $x = 5$?

decreasing (b/c $f'(5) < 0$)

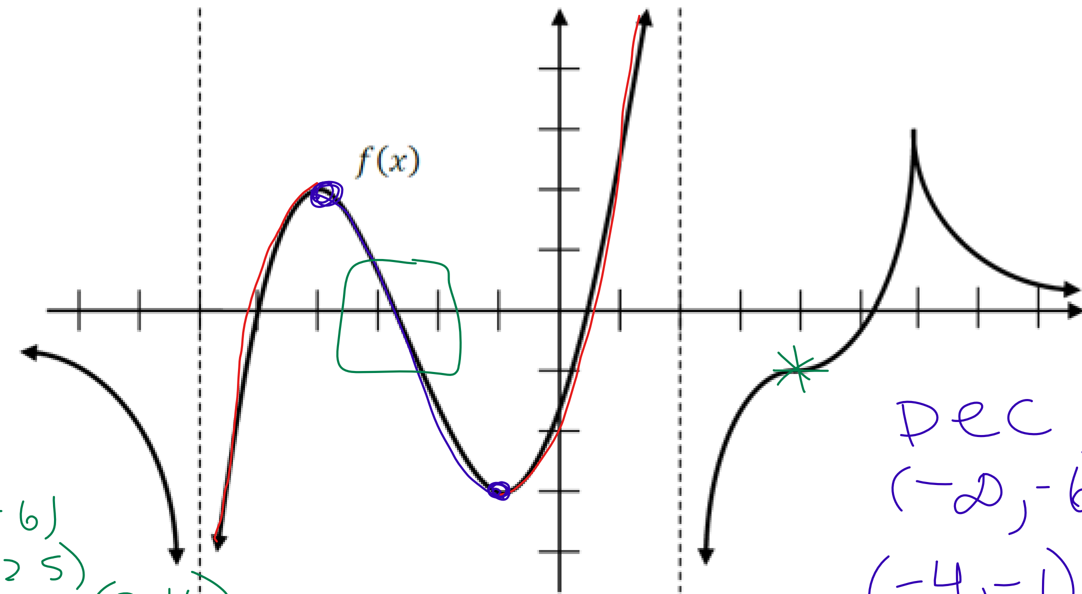
f) Is $f(x)$ concave up or concave down at $x = 4$?

$f(x)$ is CCD at $x = 4$

f) Find the x-values where $f(x)$ has a point of inflection.

$x = 2$ and 5 (b/c $f'(x)$ changes direction)

5. Use the graph of $f(x)$ below to answer the following



CCUP $(-2.5, 2)$ $(4, 6)$ $(6, \infty)$

CCdown $(-\infty, -6)$ $(-6, -2.5)$ $(2, 4)$

Dec $(-\infty, -6)$ $(-4, -1)$ $(6, \infty)$

a) Find the intervals where $f(x)$ is increasing and decreasing.

Inc $(-6, -4)$ $(-1, 2)$ $(2, 6)$

b) Find all x-values where the slope of $f(x)$ is zero.

$x = -4, -1, 4$

c) Find all x-values where the derivative of $f(x)$ does not exist

$x = -6, 2, 6$

d) Find all critical points of $f(x)$

$x = -4, -1, 4, 6$

e) Find all coordinates where $f(x)$ has relative extrema.

at $x = -4, -1, 6$

f) Find all x-values where $f(x)$ has a point of inflection (approximate if necessary)

$x = -2.5, 4$

g) Find the intervals where $f(x)$ is concave up and concave down (approximate if necessary)

6. Find the pts of horizontal and vertical tangency $y = \sqrt{9 - x^2}$.

$f'(x) = 0$

$f'(x)$ is undefined

$$\frac{dy}{dx} = \frac{1}{2}(9 - x^2)^{-\frac{1}{2}}(-2x)$$

$$\frac{dy}{dx} = \frac{-\cancel{2}x}{\cancel{2}\sqrt{9 - x^2}}$$

$\rightarrow$ if num $= 0$, hor tang $x = 0$

pt of H T is $(0, 3)$

if den $= 0$, vert tang $x = \pm 3$

pt of V T

$(-3, 0)$

$(3, 0)$

7. $f(x) = e^{2x}$ Find equation of tangent line to $f(x)$ at $\left(\frac{1}{2}\ln 5, 5\right)$

$f'(x)$, x, y

$f'(x) = e^{2x} \cdot 2$

$f'\left(\frac{1}{2}\ln 5\right) = e^{\cancel{2}\left(\frac{1}{\cancel{2}}\ln 5\right)} \cdot 2$

$= 5 \quad 2$

$= 10$

$$\boxed{y - 5 = 10\left(x - \frac{1}{2}\ln 5\right)}$$

8. Use the information in the table about $f(x)$ over $[-3, 6]$ to answer the following questions.

x	-3	$-3 < x < 0$	0	$0 < x < 3$	3	$3 < x < 6$	6
f	-4	$-$ below x-axis	0 on x-axis	$-$ below x-ax	-2	$-$ below x	0
f'	10	$+$ increasing	rel max 0 horiz tang	$-$ decreasing	DNE not differe	$+$ Increasing	2
f''	$-$	$-$ concave	$-$ down	$-$	DNE	$-$	$-$

a) Find the points of relative and absolute extrema for $f(x)$. Justify your response.

abs min $= -4$

rel max at $x = 0$ $\quad f'(x)$ + to $-$

abs max $= 0$

rel min at $x = 3$ $\quad f'(x)$ $-$ to +

b) Sketch a graph of $f(x)$ on the axes below.