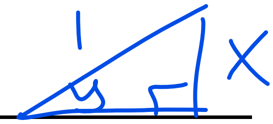
I don't know $\frac{d}{dx}\sin^{-1}x$

$a^2+x^2=1^2$

$a^2=1-x^2$



$a=\sqrt{1-x^2}$

AB Calculus: Derivatives of Inverse Functions **Name:** ______________________________

Example 1: Suppose $y=\sin^{-1}x$. Find $\frac{dy}{dx}$ using implicit differentiation.

(1) $\sin y = x$

(2) $\cos y \frac{dy}{dx} = 1$

(3) $\boxed{\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}}$

Derivatives of Inverse Trig Functions where u is a function of x

chain rule

$\boxed{\frac{d}{dx}[\sin^{-1}u] = \frac{1}{\sqrt{1-u^2}}\cdot u'}$	$\frac{d}{dx}[\cos^{-1}u] = \frac{-1}{\sqrt{1-u^2}}\cdot u'$
$\frac{d}{dx}[\tan^{-1}u] = \frac{1}{1+u^2}\cdot u'$	$\frac{d}{dx}[\cot^{-1}u] = \frac{-1}{1+u^2}\cdot u'$
$\frac{d}{dx}[\sec^{-1}u] = \frac{1}{\lvert u\rvert\sqrt{u^2-1}}\cdot u'$	$\frac{d}{dx}[\csc^{-1}u] = \frac{-1}{\lvert u\rvert\sqrt{u^2-1}}\cdot u'$

Notes:

- Domains are restricted to make them functions so do not worry about $\sin^{-1}x$ versus $\mathrm{Sin}^{-1}x$.
- $\sin^{-1}x$ and $\arcsin x$ are the same thing. Both refer to the inverse sine function.

Example 2: Find $\frac{d}{dt}[\sin^{-1}(t^2)]$

$= \frac{1}{\sqrt{1-(t^2)^2}}\cdot 2t = \frac{2t}{\sqrt{1-t^4}}$

Example 3: Find $\frac{d}{dx}[\tan^{-1}(\sqrt{x-1})]$

$\tan^{-1}x \rightarrow \frac{1}{1+x^2}$

$\sqrt{x} \rightarrow \frac{1}{2\sqrt{x}}$

$x-1 \rightarrow 1$

$\Rightarrow \frac{1}{1+(\sqrt{x-1})^2}\cdot\frac{1}{2\sqrt{x-1}}\cdot 1$

$\Rightarrow \frac{1}{1+x-1}\cdot\frac{1}{2\sqrt{x-1}} = \frac{1}{2x\sqrt{x-1}}$

Example 4: Find $\frac{d}{dx}[x\sec^{-1}(3x)]$

$= 1\sec^{-1}(3x) + x\frac{1}{\lvert 3x\rvert\sqrt{(3x)^2-1}}\cdot 3$

$= \sec^{-1}(3x) + \frac{1}{\sqrt{9x^2-1}}$

Given $y=x^2+1$, find it's inverse

(1) $x=y^2+1$ $\quad y^2=x-1$

$y=\sqrt{x-1}$

(a) What is the slope of the function? (b) the inverse?

$2x$ $\qquad \frac{1}{2\sqrt{x-1}} = \frac{1}{2y}$

$f(x) = x^3 + 2x + 1$, find $(f^{-1})'(1)$

input of inverse, therefore it's the ouput of $f(x)$

$1 = x^3 + 2x + 1$

$0 = x^3 + 2x$

$0 = x(x^2 + 2)$

$x = 0$ $\quad x^2 + 2 = 0 \rightarrow$ imaginary

$f'(x) = 3x^2 + 2$

$(f^{-1})'(1) = \dfrac{1}{f'(0)} = \dfrac{1}{3(0)^2 + 2} = \dfrac{1}{2}$

Derivative of the Inverse Function at (p, q)

$$(f^{-1})'(p) = \frac{1}{f'(q)}$$

The derivative of $f^{-1}(x)$ at the point (p, q) is the reciprocal of the derivative of $f(x)$ at the point (q, p).

Example 6 Let $f(x) = x^5 + 2x - 1$. Find of the inverse derivative $(f^{-1})'(-1)$ at the point $(0, -1)$ using ~~both~~ the logical method.

at input of inverse $= -1 \rightarrow$ tells me that output of normal function is -1

$(f^{-1})'(-1) = \dfrac{1}{f'(0)}$

$-1 = x^5 + 2x - 1$

$0 = x^5 + 2x$

$0 = x(x^4 + 2) \rightarrow x = 0$

$f'(x) = 5x^4 + 2$

$f'(0) = 5(0)^4 + 2$

$= 2$

$(f^{-1})'(-1) = \dfrac{1}{2}$

Example 7 Let $f(x) = x^3 + 2x - 1$. Find $\left.\dfrac{df^{-1}}{dx}\right|_{x=2}$ using both methods. You can use your calculator to help you find the missing coordinate.

input of inverse $\rightarrow$ output of $f(x)$

$f'(x) = 3x^2 + 2$

$2 = x^3 + 2x - 1$

$0 = x^3 + 2x - 3$ (x = 1 is a solution)

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 2 & -3 \\ & & 1 & 1 & 3 \\ \hline & 1 & 1 & 3 & 0 \end{array}$$

$(x-1)(x^2 + x + 3) = 0$

$b^2 - 4ac$

$1 - 4(1)(3) < 0$ so imaginary

answer $\dfrac{1}{f'(1)} = \dfrac{1}{3(1)^2 + 2} = \dfrac{1}{5}$