

1. Find the particular solution  $y = f(x)$  using the given initial condition.

a)  $\frac{dy}{dx} = -\frac{1}{x^2} - \frac{3}{x^4} + 12$  and  $y = 3$  when  $x = 1$ .

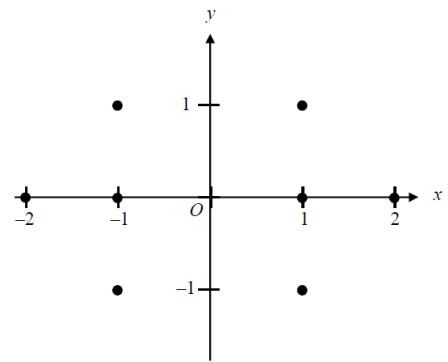
b)  $\frac{dx}{dt} = \frac{1}{t} - \frac{1}{t^2} + 6$  and  $x = 0$  when  $t = 1$ .

c)  $\frac{du}{dx} = 7x^6 - 3x^2 + 5$  and  $u = 1$  when  $x = 1$ .

d)  $\frac{dv}{dt} = 4 \sec t \tan t + e^t + 6t$  and  $v = 5$  when  $t = 0$ .

2. [No Calculator] Consider the differential equation  $\frac{dy}{dx} = \frac{y}{x}$ , where  $x \neq 0$ .

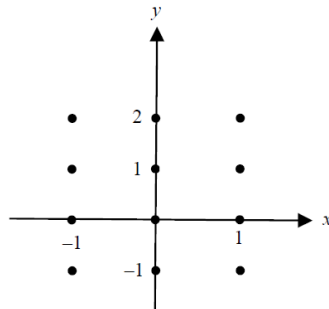
a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.



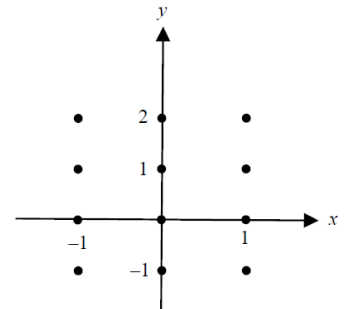
b) Find the particular solution  $y = f(x)$  to the differential equation with the initial condition  $f(-1) = 1$  and state its domain.

3. Construct a slope field for each differential equation. Draw tiny segments through the twelve lattice points shown in the graph.

a)  $\frac{dy}{dx} = 2y$



b)  $\frac{dy}{dx} = \frac{x}{2y}$



For each slope field above, sketch the solution curve that passes through the point  $(0, 1)$ .

4. Answer the following questions.

a) Given the differential equation  $\frac{dy}{dx} = x + 2$  and  $y(0) = 3$ . Find an approximation for  $y(1)$  by using Euler's method with two equal steps. Sketch your solution.

b) Solve the differential equation  $\frac{dy}{dx} = x + 2$  with the initial condition  $y(0) = 3$ , and use your solution to find  $y(1)$ .

c) The error in using Euler's Method is the difference between the approximate value and the exact value. What was the error in your answer? How could you produce a smaller error using Euler's Method?

5. Suppose a continuous function  $f$  and its derivative  $f'$  have values that are given in the following table. Given that  $f(2) = 5$ , use Euler's Method with two steps of size  $\Delta x = 0.5$  to approximate the value of  $f(3)$ .

$x$	2.0	2.5	3.0
$f'(x)$	0.4	0.6	0.8
$f(x)$	5		