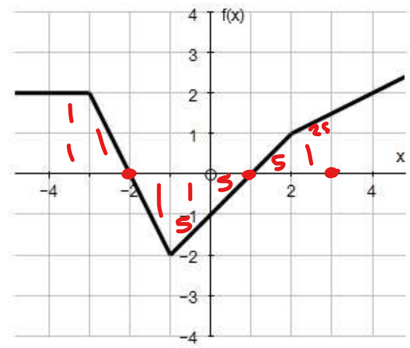
AP Calculus
5.4 Worksheet Day 2

1. Let $w(x)=\int_1^x f(t)\,dt$. The graph of $f(x)$ is shown below.



$*$ $\int_{-4}^{1} f(t)\,dt = 3+(-3) = 0$

a) Find $w(1)$

$w(1)=\int_1^1 f(t)\,dt = 0$

b) Find $w(3)$

$w(3)=\int_1^3 f(t)\,dt = 1.75$

c) Find $w(-2)$

$w(-2)=\int_1^{-2} f(t)\,dt = 3$

d) Find $w(-4)$

$w(-4)=\int_1^{-4} f(t)\,dt = -3+3=0$

e) What is $w'(x)$?

$f(x)$

f) Find $w'(2)$

$w'(2)=f(2)= 1$

g) $w'(-1)$

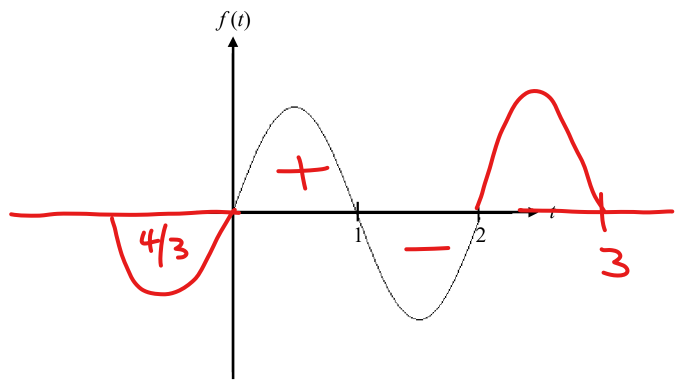
$w'(-1)=f(-1) = -2$

2. Let $F(x)=\int_0^x f(t)\,dt$. The graph of $f(t)$ given below has <u>odd symmetry</u> and is periodic (with <u>period = 2</u>). If you know that $\int_0^1 f(t)\,dt=\frac{4}{3}$, complete the following table:

x	*F(x)*
–1	4/3
0	0
1	4/3
2	0
3	4/3

$\int_0^2 f(t)\,dt = 0$

$\int_0^{-1} f(t)\,dt$



3. If a is a constant and $g(x)=\int_a^x w(t)\,dt$, what is $g'(x)$? $w(x)$

4. If a is a constant and $g(x)=\int_x^a w(t)\,dt$, what is $g'(x)$? $-w(x)$

5. Find $\frac{d}{dx}\left[\int_{-3}^{x}\sqrt{1+e^{5t}}\,dt\right]$.

$\sqrt{1+e^{5x}}$

6. If $y=\int_0^x \left(t^3-t\right)^5 dt$, find y'.

$(x^3-x)^5$

7. $k(x)=\int_{-\pi}^{x}\frac{2-\sin u}{3+\cos u}\,du$. Find $k'(x)$.

$$\frac{2-\sin x}{3+\cos x}$$

8. Find $\frac{d}{dx}\left[\int_{x}^{7}\sqrt{2p^4+p+1}\,dp\right]$

$$-\sqrt{2x^4+x+1}$$

9. What is the linearization of $f(x)=\int_{\pi}^{x}\cos^3 t\,dt$ at $x=\pi$?

TL. $x, y, \frac{dy}{dx}$

$x=\pi$

$y=f(x)=f(\pi)=\int_{\pi}^{\pi}\cos^3 t\,dt=0$

$\frac{dy}{dx}=f'(x)=\cos^3 x \quad f'(\pi)=\cos^3(\pi)=-1$

$y-0=-1(x-\pi)$

Linearization $(L(x))$

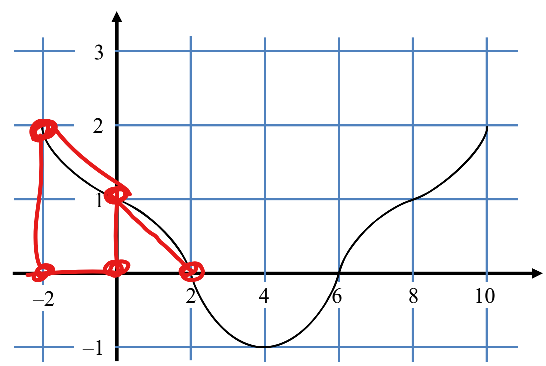
$$\boxed{L(x)=-1(x-\pi)+0}$$

10. The graph of a differentiable function f on the interval [–2, 10] is shown in the figure below. The graph of f has a horizontal tangent line at $x = 4$.

Let $h(x)=9+\int_{4}^{x}f(t)\,dt$ for $-2<x<10$.

$h'(x)=f(x)$

Graph of f



a) Find $h(4)$, $h'(4)$, and $h''(4)$

$h(4)=9+\int_{4}^{4}f(t)\,dt=9+0=9$

$h'(4)=f(4)=-1$

$h''(4)=f'(4)=0$

b) On what intervals is $\boxed{h}$ increasing? Justify your answer.

?

$h'>0$

$(-2,2) \quad (6,10) \quad f>0$

c) On what intervals is $\boxed{h}$ concave downward? Justify your answer.

$h''<0 \quad h''(x)=f'(x)$

$(-2,4)$ f is decreasing

d) Find the Trapezoidal Sum to approximate $\int_{-2}^{10}f(x)\,dx$ using 6 subintervals of length = 2.

$\left(\frac{1}{2}\right)(2+1)(2)$

$+\left(\frac{1}{2}\right)(1+0)(2)$

$\left(\frac{1}{2}\right)(2)(2+2(1+0-1+0+1)+2)$

$$\boxed{=6}$$

11. If $q(x)$ and $p(x)$ are differential functions of x and $g(x)=\int_{q(x)}^{p(x)} w(t)\,dt$, what is $g'(x)$?

$g'(x) = w(p(x))\,p'(x) - w(q(x))\,q'(x)$

12. Find $\frac{d}{dx}\left[\int_{1}^{\sin x} \sqrt{1+t^3}\,dt\right]$

$= \sqrt{1+\sin^3 x} \cdot \cos x$

13. Find $\frac{d}{dx}\left[\int_{x^2}^{x^3} \cos(2t)\,dt\right]$

$\cos(2x^3)\,3x^2 - \cos(2x^2)\,2x$

14. If $y=\int_{3x^2}^{10} \ln(2+u^2)\,du$, find y'.

$-\ln(2+(3x^2)^2)(6x)$

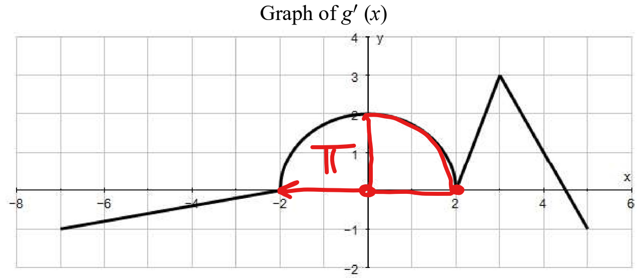
15. Find $\frac{d}{dx}\left[\int_{\sin x}^{x^3} e^{t^2}\,dt\right]$

$e^{x^6}\,3x^2 - e^{\sin^2 x}\cos x$

16. The function g is define and differentiable on the closed interval $[-7, 5]$ and satisfies $\boxed{g(0) = 5}$ → initial condition. The graph of $g'(x)$, the derivative of g, consists of a semicircle and three line segments as shown in the figure.

$A_{circle} = \pi(2)^2 = \underline{4\pi}$

Graph of $g'(x)$



a) Write an expression for $g(x)$.

$g(x) = g(0) + \int_0^x g'(t)\,dt$

b) Use your expression to find $g(3)$ and $g(-2)$.

$g(3) = g(0) + \int_0^3 g'(t)\,dt = 5 + \pi + 1.5$

$g(-2) = g(0) + \int_0^{-2} g'(t)\,dt = 5 - \pi$

c) Find the x-coordinate of each point of inflection of the graph of $g(x)$ on the interval $(-7, 5)$. Explain your reasoning.

$g''(x)$ is positive $(-7,-2)\ (-2,0)\ (2,3)$

$g''(x)$ is negative $(0,2)\ (3,5)$

x-coor of p.o. inflections → $x = 0, 2, 3$ g'' changes signs