ABCALC FTOC and USub Review Solutions

Station 1: First FToC

1. Evaluate each of the following integrals.

a) $\int_{-1}^{2}(x^3 - 3x)dx$

$\frac{1}{4}x^4 - \frac{3}{2}x^2 \Big|_{-1}^{2}$

$(4-6)-\left(\frac{1}{4}-\frac{3}{2}\right)$

$-2 + \frac{+5}{4}$

$\boxed{\frac{-3}{4}}$

b) $\int_{1}^{8}\sqrt[3]{x}dx$

$\int_{1}^{8} x^{\frac{1}{3}}dx$

$\frac{3}{4}x^{\frac{4}{3}}\Big|_{1}^{8}$

$\frac{3}{4}(16) - \frac{3}{4}$

$\boxed{\frac{45}{4} \text{ or } 11.25}$

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c) $\int_1^9 \frac{1}{3x}\,dx$

$\frac{1}{3}\ln|x| \Big|_1^9$

$\frac{1}{3}\ln|9| - \frac{1}{3}\ln|1|$

$\boxed{\frac{1}{3}\ln 9}$

d) $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{6}{\sqrt{1-t^2}}\,dt$

$6\sin^{-1}t \Big|_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}$

$6\left(\frac{\pi}{3}\right) - 6\left(\frac{\pi}{6}\right)$

$2\pi - \pi$

$\boxed{\pi}$

e) $\int (e^x + 9^x + \csc x \cot x)\,dx$

$e^x + \frac{9^x}{\ln 9} - \csc x + C$

f) $\int \left(x^{\frac{4}{5}} + \frac{1}{\sqrt[4]{x}} - \sqrt{x}\right)dx$

$\frac{5}{9}x^{\frac{9}{5}} + \frac{4}{3}x^{\frac{3}{4}} - \frac{2}{3}x^{\frac{3}{2}} + C$

g) $\int (3\sin x + 4\cos x - \sec^2 x)\,dx$

$-3\cos x + 4\sin x - \tan x + C$

h) $\int (5 + x\sqrt{x}\,)\,dx$

$\int (5 + x^{\frac{3}{2}})\,dx$

$5x + \frac{2}{5}x^{\frac{5}{2}} + C$

Station 2: Second FToC

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1. Find the derivative of each of the following.

a) $\int_0^x \left(\frac{t^2}{1+t^3}\right) dt$

$\frac{x^2}{1+x^3}$

b) $\int_x^1 \sqrt{t+\sin t}\, dt$

$-\sqrt{x+\sin x}$

c) $\int_{\sqrt{x}}^x \left(\frac{e^t}{t}\right) dt$

$\frac{e^x}{x} - \frac{e^{\sqrt{x}}}{\sqrt{x}}\left(\frac{1}{2\sqrt{x}}\right)$

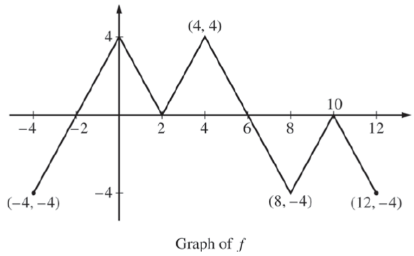
$\frac{e^x}{x} - \frac{e^{\sqrt{x}}}{2x}$

d) $\int_{2x}^{3x+1} \sin(t^4)\, dt$

$\sin((3x+1)^4)(3) - \sin((2x)^4)(2)$

$3\sin((3x+1)^4) - 2\sin(16x^4)$

2.



Graph of f

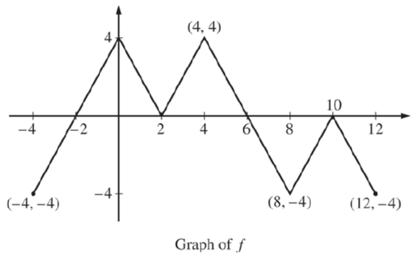
$g(x)$ $F(x)$ area
$g'(x)$ $f(x)$ graph
$g''(x)$ $f'(x)$ slope

The figure above shows the graph of the piecewise-linear function f. For $-4 \le x \le 12$, the function g by $g(x) = \int_2^x f(t)\,dt$.

(a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer

neither since g' does not change sign at $x=10$

2.



Graph of f

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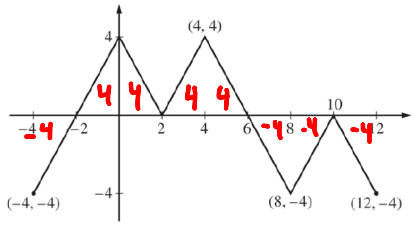
The figure above shows the graph of the piecewise-linear function f. For $-4 \le x \le 12$, the function g by $g(x) = \int_2^x f(t)\,dt$.

(b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.

yes since $g''(x)$ changes sign at $x=4$

2.



Graph of f

$g(x)$	$F(x)$	area
$g'(x)$	$f(x)$	graph
$g''(x)$	$f'(x)$	slope

The figure above shows the graph of the piecewise-linear function f. For $-4 \le x \le 12$, the function g by $g(x) = \int_2^x f(t)\,dt$.

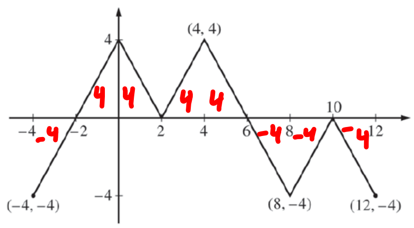
(c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \le x \le 12$. Justify your answers.

x	y
-4	$g(-4) = -4$
-2	$g(-2) = -8$
6	$g(6) = 8$
12	$g(12) = -4$

abs min is -8 at $x = -2$

abs max is 8 at $x = 6$

2.



Graph of f

$g(x)$	$F(x)$	area
$g'(x)$	$f(x)$	graph
$g''(x)$	$f'(x)$	slope

The figure above shows the graph of the piecewise-linear function f. For $-4 \le x \le 12$, the function g by $g(x) = \int_2^x f(t)\,dt$.

(d) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.

$[-4, 2] \cup [10, 12]$

Station 3: U Substitution

1. Evaluate each of the following integrals.

a) $\int (1-x)^9 dx$

$u = 1-x \quad du = -dx \quad dx = -du$

$\int u^9 \cdot -du$

$-\frac{1}{10} u^{10} + C$

$\boxed{\frac{-1}{10}(1-x)^{10} + C}$

b) $\int \sin^4 x \cos x \, dx$

$u = \sin x \quad du = \cos x \, dx$

$dx = \frac{du}{\cos x}$

$\int u^4 \cdot \cos x \cdot \frac{du}{\cos x}$

$\int u^4 du = \frac{1}{5} u^5 + C$

$\boxed{\frac{1}{5} \sin^5 x + C}$

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c) $\int e^{4\cot x} \csc^2 x \, dx$

$u = 4\cot x \quad du = -4\csc^2 x \, dx$

$dx = \frac{du}{-4\csc^2 x}$

$\int e^u \cdot \csc^2 x \cdot \frac{du}{-4\csc^2 x} = \frac{-1}{4}\int e^u du$

$\frac{-1}{4}e^u + C \rightarrow \boxed{\frac{-1}{4}e^{4\cot x} + C}$

d) $\int \frac{\tan^{-1} x}{1+x^2} dx$

$u = \tan^{-1} x \quad du = \frac{1}{1+x^2} dx$

$dx(1+x^2) = du$

$\int \frac{u}{1+x^2} \cdot (1+x^2) du$

$\int u \, du$

$\frac{1}{2}u^2 + C \rightarrow \boxed{\frac{1}{2}(\tan^{-1} x)^2 + C}$

e) $\int_0^1 x^2 \cos(x^3) \, dx$

$u = x^3 \quad du = 3x^2 dx$

$u(1) = 1 \quad dx = \frac{du}{3x^2}$

$u(0) = 0$

$\int_0^1 x^2 \cos u \cdot \frac{du}{3x^2} \quad \frac{1}{3}\int_0^1 \cos u \, du$

$\frac{1}{3}\sin u \Big|_0^1 = \boxed{\frac{1}{3}\sin(1)}$

f) $\int_1^{10} \frac{x}{x^2-4} dx$

$u = x^2 - 4 \quad du = 2x \, dx$

$u(10) = 96 \quad dx = \frac{du}{2x}$

$u(1) = -3$

$\int_{-3}^{96} \frac{x}{u} \cdot \frac{du}{2x}$

$\frac{1}{2}\int_{-3}^{96} \frac{1}{u} du$

$\frac{1}{2}\left[\ln|u| \Big|_{-3}^{96}\right] = \frac{1}{2}\ln 96 - \ln 3$

$\boxed{\frac{1}{2}\ln 32}$

Station 4: Algebraic Techniques

1. Evaluate each of the following integrals.

a) $\int \frac{\sqrt{x} - 2x^2}{x} dx$

$\int (x^{-\frac{1}{2}} - 2x) dx$

$2x^{\frac{1}{2}} - x^2 + c$

$\boxed{2\sqrt{x} - x^2 + c}$

b) $\int \frac{x^2 + x + 1}{x^2 + 1} dx$

$$\begin{array}{r|l} & 1 \\ x^2+0x+1 & x^2+x+1 \\ & -x^2-0x-1 \\ \hline & x \end{array}$$

$\int \left(1 + \frac{x}{x^2+1}\right) dx \rightarrow x + \int \frac{x}{x^2+1} dx$

$u = x^2+1 \quad du = 2x\,dx$

$dx = \frac{du}{2x}$

$\int \frac{x}{u} \cdot \frac{du}{2x}$

$\int \frac{1}{2} \cdot \frac{1}{u} du$

$\frac{1}{2} \ln|u| + C$

$\boxed{x + \frac{1}{2} \ln|x^2+1| + C}$

c) $\int \left(\frac{1}{x}+1\right)^2 dx$ $\quad \left(\frac{1}{x}+1\right)\left(\frac{1}{x}+1\right)$

$\int \left(\frac{1}{x^2}+\frac{2}{x}+1\right)dx \rightarrow \int \left(x^{-2}+\frac{2}{x}+1\right)dx$

$\boxed{\frac{-1}{x}+2\ln|x|+x+C}$

d) $\int \frac{3}{x^2+2x+2}dx$

$\left(\frac{2}{2}\right)^2=1$

$\int \frac{3}{x^2+2x+1-1+2}dx$

$\int \frac{3}{(x+1)^2+1}dx \quad u=x+1 \quad du=dx$

$\int \frac{3}{u^2+1}\cdot du \rightarrow 3\tan^{-1}u+C$

$\boxed{3\tan^{-1}(x+1)+C}$

e) $\int \frac{1}{\sqrt{49-x^2}}dx$ $\quad \int \frac{1}{\sqrt{49\left(1-\frac{x^2}{49}\right)}}dx$

$\frac{1}{7}\int \frac{1}{\sqrt{1-\left(\frac{x}{7}\right)^2}}dx$ $\quad u=\frac{1}{7}x$

$du=\frac{1}{7}dx$

$dx=7du$

$\frac{1}{7}\int \frac{1}{\sqrt{1-u^2}}\cdot 7du = \sin^{-1}u+C$

$\boxed{\sin^{-1}\left(\frac{x}{7}\right)+C}$

f) $\int \frac{4}{4x^2+36}dx$ $\quad \frac{4x^2}{36}=\frac{x^2}{9}$

$4\int \frac{1}{36\left(\frac{4x^2}{36}+1\right)}dx$

$\frac{1}{9}\int \frac{1}{\left(\frac{x}{3}\right)^2+1}dx \quad u=\frac{x}{3} \quad du=\frac{1}{3}dx$

$dx=3du$

$\frac{1}{9}\int \frac{1}{u^2+1}\cdot 3du$

$\frac{1}{3}\tan^{-1}u+C$

$\boxed{\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)+C}$

Station 5: Intro to Definite Integrals

t hours	$R(t)$ gallons/hour
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above shows the rate measured every 3 hours for a 24-hour period.

a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)dt$.

b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}(768 + 23t - t^2)$. Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period. Indicate units of measure.

a) $6(10.4) + 6(11.2) + 6(11.3) + 6(10.2) = 6(43.1) = 258.6$ gallons

b) $\frac{f(24)-f(0)}{24-0} = \frac{9.6-9.6}{24} = 0$ yes, by MVT

c) $\frac{1}{24}\int_0^{24} Q(t)dt \approx 10.785$ gallons/hr

x	2	5	7	8
$f(x)$	10	30	40	20

The function f is continuous on the closed interval [2, 8] and has values that are given in the table above. Using the subintervals [2, 5], [5, 7], and [7, 8], what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$?

$$\int_2^8 f(x)\,dx \approx \frac{1}{2}(3)(10+30) + \frac{1}{2}(2)(30+40) + \frac{1}{2}(1)(40+20)$$

$$= 60 + 70 + 30 = \boxed{160}$$