AB Calculus Fundamental Theorem of Calculus Day 1 Notesheet **Name: ___________________________**

To Begin: Answer all parts of the following two questions.

1. Find the derivative of the following

a) $f(x) = \frac{1}{3}x^3 + 5$

$f'(x) = x^2$

b) $f(x) = (x + \sin x)$

$f'(x) = 1 + \cos x$

c) $f(x) = \frac{1}{\ln 3}3^x$

$f'(x) = \frac{1}{\ln 3} 3^x \ln 3 = 3^x$

d) $f(x) = \ln x + \sqrt{x} + 6x + 7$

$f'(x) = \frac{1}{x} + \frac{1}{2\sqrt{x}} + 6$

e) $f(x) = e^x + \tan^{-1} x + 2$

$f'(x) = e^x + \frac{1}{1+x^2}$

f) $f(x) = \tan x + \sec x$

$f'(x) = \sec^2 x + \sec x \tan x$

2. The position of a particle moving along a line is given by the function $s(t) = -t^3 + 6t^2 - 2$

a) Find the change in position from time t = 0 to time t = 2.

$s(2) - s(0) = 14 - (-2) = 16$

b) Find an equation for the velocity of the particle. Find the velocity of the particle at time t = 2.

$v(t) = s'(t) = -3t^2 + 12t$ $v(2) = 12$

c) Find an equation for the acceleration of the particle. Find the particle's acceleration at time t = 2.

$a(t) = v'(t) = s''(t) = -6t + 12$ $a(2) = 0$

d) Find the average rate of change of the position function (average velocity) of the particle from time t = 0 to time t = 2.

$\frac{s(2) - s(0)}{2 - 0} = \frac{16}{2} = 8$

e) Integrate the velocity function over the interval [0, 2]. What do you notice? (Hint: look at other answers)

$\int_0^2 (-3t^2 + 12t)\,dt = \left[-t^3 + 6t^2\right]_0^2 = \left[-(2)^3 + 6(2)^2\right] - \left[-(0)^3 + 6(0)^2\right]$

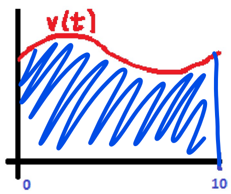
$[-8 + 24] - [0] = 16$

f) Use your calculator to find the average value of the velocity function over the interval [0, 2]. What do you notice? (Hint: look at other answers)

$\frac{\int_0^2 v(t)\,dt}{2 - 0} = \frac{16}{2} = 8$

The Fundamental Theorem of Calculus (FTOC) has two parts. These two parts tie together the concept of integration and differentiation and is regarded by some to be the most important computational discovery in the history of mathematics! Today we will look at the first part, the evaluation part.

Ex 1: The following graph shows the velocity function for a particle moving along a line. Given the following write an expression that will give you the change in position (displacement) of the particle.



$$\int_0^{10} v(t)\,dt = \Big[s(t)\Big]_0^{10} = s(10) - s(0)$$

v(t) is the derivative of s(t) OR s(t) is the antiderivative of v(t)

Ex 2: Suppose a car's position is given by $s(t) = \frac{3}{2}t^2 + 30t + 25$ where t is time in seconds, and $0 \le t \le 10$.

a) What is the position of the car at t = 0 seconds?

$s(0) = 25$

b) What is the position of the car at t = 10 seconds?

$s(10) = 475$

c) What is the change in position of the car from time t = 0 to time t = 10 seconds?

$s(10) - s(0) = 475 - 25 = 450$

d) How does this question relate to the previous example?

change in position "displacement" $= s(b) - s(a) = \int_a^b v(t)\,dt$

The Fundamental Theorem of Calculus [The Evaluation Part]

If f is continuous at every point of [a, b],

$\int_a^b f(x)dx = F(b) - F(a)$,

Where F(x) is the antiderivative of f(x).

Integral Formulas Part 1: Constants and Power Rule

$\int a\,dx = ax + C$, where a is a constant (→ the constant of integration)

When $n \neq -1$, $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C$

When integrating a constant, just put a variable on it. For functions that you can write as a power x, add 1 to the exponent and multiply by the reciprocal of the new exponent.

Ex 3: Evaluate the following Integrals

a) $\int_2^5 3\,dx = [3x]_2^5$

$= 3(5) - 3(2)$

$15 - 6 = \boxed{9}$

b) $\int_3^6 dx \quad x\Big|_3^6 = 6 - 3 = \boxed{3}$

c) $\int_0^3 x^2 dx \quad \frac{1}{3}x^3\Big|_0^3$

$\frac{1}{3}(3)^3 - \frac{1}{3}(0)^3 = \boxed{9}$

d) $\int_{-1}^8 x^{\frac{-2}{3}} dx \quad 3x^{\frac{1}{3}}\Big|_{-1}^8$

$3(8)^{\frac{1}{3}} - 3(-1)^{\frac{1}{3}} = 3(2) - 3(-1)$

$= 6 + 3 = \boxed{9}$

$5\int_0^2 x^3 dx \rightarrow$ e) $\int_0^2 5x^3 dx \quad \frac{5x^4}{4}\Big|_0^2$

$\frac{5}{4}(2)^4 - \frac{5}{4}(0)^4 = \boxed{20}$

$\frac{5}{4}(2^4 - 0^4)$

f) $\int_4^9 \frac{1}{\sqrt{x}} dx = \int_4^9 x^{-\frac{1}{2}} dx$

$= 2x^{\frac{1}{2}}\Big|_4^9$

$= 2(9)^{\frac{1}{2}} - 2(4)^{\frac{1}{2}} = 2(3) - 2(2)$

$= 6 - 4 = \boxed{2}$

Note: Only need to add + C for indefinite integrals (integrals with no bounds)

Integral Formulas Part 2: Logs and Exponentials

When $n = -1$, $\int \frac{1}{x} dx = \ln|x| + C$

$\int e^x\,dx = e^x + C$

$\int a^x\,dx = \frac{a^x}{lna} + C$, where a is a constant

Ex 4: Evaluate the following integrals.

a) $\int_3^6 \frac{3}{x} dx = 3\int_3^6 \frac{1}{x} dx$

$3\ln|x|\Big|_3^6 = 3\ln 6 - 3\ln 3$

$= 3(\ln 6 - \ln 3)$

$= 3\ln\left(\frac{6}{3}\right) = 3\ln 2 = \ln 2^3 = \boxed{\ln 8}$

b) $\int_{-1}^1 2e^x dx = 2e^x\Big|_{-1}^1$

$= 2e^1 - 2e^{-1} = 2e - \frac{2}{e}$

c) $\int_{-1}^2 3^x dx \quad \frac{1}{\ln 3} 3^x\Big|_{-1}^2$

$= \frac{1}{\ln 3}\cdot 3^2 - \frac{1}{\ln 3}(3)^{-1} = \boxed{\frac{9}{\ln 3} - \frac{1}{3\ln 3}}$

d) $\int_1^4 \frac{1}{x^2} dx = \int_1^4 x^{-2} dx$

$= \frac{1}{-1}x^{-1}\Big|_1^4$

$= (-1)(4)^{-1} - (-1)(1)^{-1}$

$-\frac{1}{4} + 1 = \frac{3}{4}$

Integral Formulas Part 3: Trig Functions

$\int \sin x\,dx = -\cos x + C$	$\int \cos x\,dx = \sin x + C$
$\int \sec^2 x\,dx = \tan x + C$	$\int \csc^2 x\,dx = -\cot x + C$
$\int \csc x \cot x\,dx = -\csc x + C$	$\int \sec x \tan x\,dx = \sec x + C$

Ex 5: Evaluate the following integrals.

a) $\int_{\frac{\pi}{2}}^{\pi} (1 + \cos x)dx$

$\left[x + \sin x\right]_{\frac{\pi}{2}}^{\pi}$

$(\pi + \sin\pi) - \left(\frac{\pi}{2} + \sin\frac{\pi}{2}\right)$

$\pi + 0 - \frac{\pi}{2} - 1 = \boxed{\frac{\pi}{2} - 1}$

b) $\int_0^{\frac{\pi}{4}} \sec x \tan x\,dx$

$= \left[\sec x\right]_0^{\pi/4}$

$= \sec\frac{\pi}{4} - \sec 0$

$\boxed{= \frac{2}{\sqrt{2}} - 1}$

Integral Formulas Part 4: Inverse Trig

$\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1} x + C$	$\int \frac{1}{1+x^2}dx = \tan^{-1} x + C$
$\int \frac{1}{\lvert x\rvert\sqrt{x^2-1}}dx = \sec^{-1} x + C$	

Ex 6: Evaluate the following integrals.

a) $\int_{\frac{1}{2}}^{1} \frac{2}{\sqrt{1-x^2}}dx$

$2\sin^{-1} x \Big|_{\frac{1}{2}}^{1}$

$2\sin^{-1}(1) - 2\sin^{-1}\left(\frac{1}{2}\right)$

$2\frac{\pi}{2} - 2\frac{\pi}{6} = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

b) $\int_{-1}^{\sqrt{3}} \frac{1}{1+x^2}dx$

$\tan^{-1} x \Big|_{-1}^{\sqrt{3}}$

$\tan^{-1}\sqrt{3} - \tan^{-1}(-1)$

$\frac{\pi}{3} - \frac{-\pi}{4}$

$\frac{4}{4}\frac{\pi}{3} + \frac{\pi}{4}\cdot\frac{3}{3} = \frac{7\pi}{12}$