

AB Calculus Implicit Differentiation Practice

Name: _____

1. Find $\frac{dy}{dx}$ *implies taking the derivative w/ respect to "x"*

a) $5x^2 = -4y^3 - 3y^2 + 2$

$$10x = -12y^2 \frac{dy}{dx} - 6y \frac{dy}{dx}$$

$$10x = \frac{dy}{dx} (-12y^2 - 6y) \Rightarrow \frac{dy}{dx} = \frac{10x}{-12y^2 - 6y}$$

b) $2x^3 + 3y^2 + 4x = 5$

$$\frac{dy}{dx} = \frac{-6x^2 - 4}{6y}$$

c) $\sin y + 3 = y + \cos x$

$$(\cos y) \frac{dy}{dx} = 1 \frac{dy}{dx} - \sin x$$

$$(\cos y) \frac{dy}{dx} - \frac{dy}{dx} = -\sin x$$

$$\frac{dy}{dx} (\cos y - 1) = -\sin x$$

$$\frac{dy}{dx} = \frac{-\sin x}{\cos y - 1}$$

d) $\cos(y^3) = x^3 + 2x + 5$

$$-\sin(y^3) 3y^2 \frac{dy}{dx} = 3x^2 + 2$$

$$\frac{dy}{dx} = \frac{3x^2 + 2}{-3y^2 \sin(y^3)}$$

2. Find the slope of the curve $xy^2 + 2xy = 8$ at the point (1, 2)

$$1y^2 + 2y \frac{dy}{dx} x + 2(1 \cdot y + x \frac{dy}{dx}) = 0$$

$$1(2)^2 + 2(2) \frac{dy}{dx} 1 + 2(2 + 1 \frac{dy}{dx}) = 0$$

$$4 + 4 \frac{dy}{dx} + 4 + 2 \frac{dy}{dx} = 0 \rightarrow 8 + 6 \frac{dy}{dx} = 0 \rightarrow \frac{6 \frac{dy}{dx}}{6} = \frac{-8}{6} \rightarrow \frac{dy}{dx} = \frac{-4}{3}$$

3. Find $\frac{d^2y}{dx^2}$ if $y^2 = x^2 + 2x$

$$\frac{2y \frac{dy}{dx}}{2y} = \frac{2x + 2}{2y} \rightarrow \frac{d}{dx} \left[\frac{dy}{dx} = \frac{x+1}{y} \right] \rightarrow \frac{d^2y}{dx^2} = \frac{y(1) - (x+1) \frac{dy}{dx}}{y^2} = \frac{y - (x+1) \left(\frac{x+1}{y} \right)}{y^2}$$

4. Find the points at which the graph of $4x^2 + y^2 - 8x + 4y + 4 = 0$ has a vertical tangent line.

$$8x + 2y \frac{dy}{dx} - 8 + 4 \frac{dy}{dx} = 0$$

$$2y \frac{dy}{dx} + 4 \frac{dy}{dx} = -8x + 8$$

$$\frac{dy}{dx} (2y + 4) = -8x + 8$$

$$\frac{dy}{dx} = \frac{-8x + 8}{2y + 4}$$

$$2y + 4 = 0 \rightarrow 2y = -4 \rightarrow y = -2$$

$$4x^2 + (-2)^2 - 8x + 4(-2) + 4 = 0$$

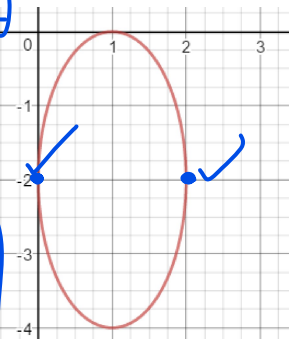
$$4x^2 + 4 - 8x - 8 + 4 = 0$$

$$4x^2 - 8x = 0$$

$$4x(x - 2) = 0$$

$$x = 0, 2$$

Pts (0, -2)
(2, -2)



5. Find the points at which the graph of $x^2 + xy + y^2 = 6$ has a horizontal tangent line.

$$2x + 1y + x \cdot \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x - y}{x + 2y}$$

$$-2x - y = 0 \rightarrow y = -2x$$

Ino g $\rightarrow x^2 + x(-2x) + (-2x)^2 = 6$

$$x^2 - 2x^2 + 4x^2 = 6$$

$$3x^2 = 6$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

$$y = -2 \pm \sqrt{2}$$

Pts $(\sqrt{2}, -2\sqrt{2})$
 $(-\sqrt{2}, 2\sqrt{2})$

