Inverse Trig Derivatives

Find the derivative of each of the following.

1. $y = \cos^{-1}\left(\frac{1}{x}\right)$

$$y' = \frac{-1}{\sqrt{1-\left(\frac{1}{x}\right)^2}} \cdot \frac{-1}{x^2}$$

$$y' = \frac{1}{x^2\sqrt{1-\frac{1}{x^2}}}$$

2. $y = \tan^{-1}(3x^2)$

$$y' = \frac{1}{1+(3x^2)^2} \cdot 6x$$

$$y' = \frac{6x}{1+9x^4}$$

3. $y = \csc^{-1}(5x-2)$

$$y' = \frac{-1}{|5x-2|\sqrt{(5x-2)^2-1}} \cdot 5$$

$$y' = \frac{-5}{|5x-2|\sqrt{(5x-2)^2-1}}$$

4. $y = \cot^{-1}(\ln(x^2))$

$$y' = \frac{-1}{1+(\ln(x^2))^2} \cdot \frac{1}{x^2} \cdot 2x$$

$$y' = \frac{-2}{x(1+(\ln(x^2))^2)}$$

5. $y = \sin^{-1}(xy)$

$$y' = \frac{1}{\sqrt{1-(xy)^2}} \cdot (xy'+y)$$

$$y'\sqrt{1-(xy)^2} = xy'+y$$

$$y'\sqrt{1-(xy)^2} - xy' = y$$

$$y'\left(\sqrt{1-(xy)^2} - x\right) = y$$

$$y' = \frac{y}{\sqrt{1-(xy)^2}-x}$$

Derivatives of Logarithmic Functions

1. $y = \ln(9x^2 - 8)$

$y' = \frac{1}{9x^2-8} \cdot 18x$

$y' = \frac{18x}{9x^2-8}$

2. $y = x^2 \ln(x)$

$y' = x^2 \cdot \frac{1}{x} + \ln x \cdot 2x$

$y' = x + 2x\ln x$

3. $y = \ln((x+1)(2x+9))$

$y = \ln(x+1) + \ln(2x+9)$

$y' = \frac{1}{x+1} + \frac{1}{2x+9} \cdot 2$

$y' = \frac{1}{x+1} + \frac{2}{2x+9}$

4. $y = (\ln(\ln x))^3$

$y' = 3(\ln(\ln x))^2 \cdot \frac{1}{\ln x} \cdot \frac{1}{x}$

$y' = \frac{3(\ln(\ln x))^2}{x \ln x}$

5. Write an equation of a line tangent to $y = log_2(1 + 4x^{-1})$ at the point where x = 4.

$y = \frac{\ln(1+\frac{4}{x})}{\ln 2}$ $\quad y(4) = \frac{\ln(2)}{\ln(2)} = 1$

$y = \frac{1}{\ln 2} \cdot \ln(1+\frac{4}{x})$

$y' = \frac{1}{\ln 2} \cdot \frac{1}{1+\frac{4}{x}} \cdot \frac{-4}{x^2}$

$y'(4) = \frac{1}{\ln 2} \cdot \frac{1}{2} \cdot \frac{-4}{16} = \frac{-1}{8\ln 2}$

$y - 1 = \frac{-1}{8\ln 2}(x-4)$

Derivatives of Exponential Functions

1. $y = 11^x$

$y' = \ln 11 \cdot 11^x$

2. $y = 7^{4x-x^2}$

$y' = \ln 7 \cdot 7^{4x-x^2} \cdot (4-2x)$

3. $y = e^{1-x}$

$y' = e^{1-x} \cdot (-1)$

$y' = -e^{1-x}$

4. $y = e^{\sqrt{2x-3}}$

$y' = e^{\sqrt{2x-3}} \cdot \frac{1}{2\sqrt{2x-3}} \cdot 2$

$y' = \frac{e^{\sqrt{2x-3}}}{\sqrt{2x-3}}$

f	d
e^x	e^x
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
$2x-3$	2

5. $xe^y - 10x + 3y = 0$

$xe^y \cdot y' + e^y - 10 + 3y' = 0$

$xe^y y' + 3y' = 10 - e^y$

$y'(xe^y + 3) = 10 - e^y$

$y' = \frac{10 - e^y}{xe^y + 3}$

Derivatives of Inverse Functions

Find $(f^{-1})(a)$ for the function f and the real number a.

1. $f(x) = x^3 + 2x - 1$, a = 2 and f(1) = 2.

orig: $(1,2)$ inv $(2,1)$

m 1

$x = y^3 + 2y - 1$

$1 = 3y^2 y' + 2y'$

$1 = 3(1)^2 y' + 2y'$

$1 = 5y'$

$y' = \frac{1}{5}$

m 2

$f'(x) = 3x^2 + 2$

$f'(1) = 3(1)^2 + 2 = 5$

$(f^{-1})'(2) = \frac{1}{5}$

Find $(f^{-1})(a)$ for the function f and the real number a.

2. $y = \sin(x), \frac{-\pi}{2} \le x \le \frac{\pi}{2},\ a = \frac{1}{2}$

orig: $\left(\frac{\pi}{6}, \frac{1}{2}\right)$ inv: $\left(\frac{1}{2}, \frac{\pi}{6}\right)$

$\frac{1}{2} = \sin x$

$x = \frac{\pi}{6}$

m1

$x = \sin y$

$1 = \cos y \cdot y'$

$1 = \cos\frac{\pi}{6} \cdot y'$

$1 = \frac{\sqrt{3}}{2} y'$

$y' = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$

m2

$y' = \cos x$

$y'\left(\frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$(f^{-1})'\left(\frac{1}{2}\right) = \frac{2}{\sqrt{3}}$

Find $(f^{-1})(a)$ for the function f and the real number a.

3. $f(x) = x^3 - \frac{4}{x}$, a = 6 and f(2) = 6.

orig: $(2,6)$ inv: $(6,2)$

m1

$x = y^3 - \frac{4}{y}$

$1 = 3y^2 y' + \frac{4}{y^2} \cdot y'$

$1 = 3(2)^2 y' + \frac{4}{2^2} y'$

$1 = 12y' + y'$

$1 = 13y'$

$y' = \frac{1}{13}$

m2

$f'(x) = 3x^2 + \frac{4}{x^2}$

$f'(2) = 3(2)^2 + \frac{4}{2^2} = 12 + 1 = 13$

$(f^{-1})'(6) = \frac{1}{13}$

Implicit Differentiation

Find the derivative of each of the following.

1. $y^3 + y^2 - 5y - x^2 = -4$

$3y^2y' + 2yy' - 5y' - 2x = 0$

$y'(3y^2 + 2y - 5) = 2x$

$y' = \frac{2x}{3y^2 + 2y - 5}$

2. $sinx + 2\cos(2y) = 1$

$\cos x - 2\sin(2y) \cdot 2y' = 0$

$-2\sin(2y) \cdot 2y' = -\cos x$

$y' = \frac{-\cos x}{-4\sin(2y)}$

$y' = \frac{\cos x}{4\sin(2y)}$

3. $sinx = x(1 + tany)$

$\cos x = x(\sec^2 y)y' + (1+\tan y)(1)$

$x(\sec^2 y)y' = \cos x - 1 - \tan y$

$y' = \dfrac{\cos x - 1 - \tan y}{x\sec^2 y}$

4. $x^3y^3 = x$

$x^3 \cdot 3y^2 y' + y^3 \cdot 3x^2 = 1$

$3x^3y^2y' = 1 - 3x^2y^3$

$y' = \dfrac{1-3x^2y^3}{3x^3y^2}$

5. $e^y = \ln(x) + sin^3(\sqrt{3x-5})$

$e^y y' = \dfrac{1}{x} + 3\sin^2(\sqrt{3x-5}) \cdot \cos(\sqrt{3x-5}) \cdot \dfrac{1}{2\sqrt{3x-5}} \cdot 3$

$y' = \dfrac{\dfrac{1}{x} + \dfrac{9\sin^2\sqrt{3x-5}\,\cos\sqrt{3x-5}}{2\sqrt{3x-5}}}{e^y}$

$y' = \dfrac{1}{xe^y} + \dfrac{9\sin^2\sqrt{3x-5}\,\cos\sqrt{3x-5}}{2\sqrt{3x-5}\cdot e^y}$