AB Calculus Information About f(x) given f'(x) Homework Name: Key

1. Complete each of the following statements.

a) When f' is + , the graph of f is increasing.

b) When f' is − , the graph of f is decreasing.

c) When f'' is + , the graph of f is concave upward.

d) When f'' is − , the graph of f is concave downward.

e) When f' is increasing , the graph of f is concave upward.

f) When f' is decreasing , the graph of f is concave downward.

2. Use the function $f(x) = 3x - x^3 + 5$ to answer the following.

a) Find the intervals that the function is increasing. Justify your response.

$f'(x) = 3 - 3x^2$ $0 = 3(1-x)(1+x)$ CN $x = 1, -1$ − | + | − at -1, 1

Inc: $(-1, 1)$ $f'(x) > 0$

$0 = 3(1-x^2)$

b) Find the intervals that the function is decreasing. Justify your response.

Dec: $(-\infty, -1)$ $(1, \infty)$ $f'(x) < 0$

c) Find the intervals that the function is concave up. Justify your response.

$f''(x) = -6x$ + | − at 0

$0 = -6x \rightarrow x = 0$

CCU: $(-\infty, 0)$ $f''(x) > 0$

d) Find the intervals that the function is concave down. Justify your response.

CCD: $(0, \infty)$ $f''(x) < 0$

e) Find the x-coordinates of all points of inflection for the function. Justify your response.

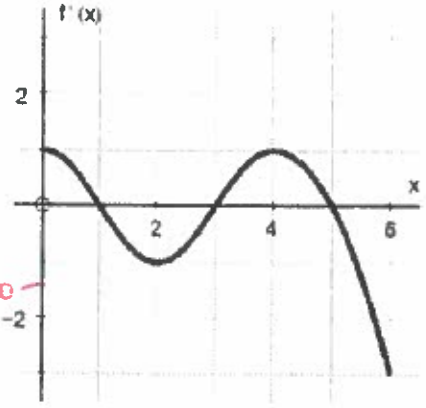
$x = 0$ $f''(x)$ changes signs

f) Find all relative extrema and label them as a maximum or minimum. Justify your response.

@ $x = -1$ is a relative min $f'(x)$ changes − to +

@ $x = 1$ is a relative max $f'(x)$ changes + to −

3. Use the graph of $f'(x)$ defined over the interval $[0, 6]$ provided below to answer the following.



a) When is $f(x)$ increasing? When is $f(x)$ decreasing? Justify your responses.

Inc: $(0,1)$ $(3,5)$ $f'(x) > 0$

Dec: $(1,3)$ $(5,6)$ $f'(x) < 0$

b) Determine the x-coordinates of all local extrema. Justify your response.

rel max @ $x=1$, $x=5$ $f'(x)$ changes + to −

rel min @ $x=3$ $f'(x)$ changes − to +

c) When is f concave up? When is f concave down? Justify your responses.

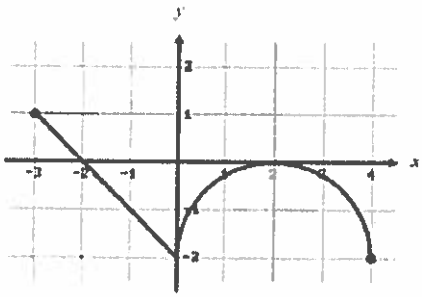
CCU: $(2,4)$ $f'(x)$ increasing $(f''(x) > 0)$

CCD: $(0,2)$ $(4,6)$ $f'(x)$ decreasing $(f''(x) < 0)$

d) Find the x-coordinates of all points of inflection. Justify your response.

$x=2$, $x=4$ $f'(x)$ changes direction ($f''(x)$ changes signs)

4. Let f be a function defined on the closed interval $[-3, 4]$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown below.



Graph of f'

a) When is $f(x)$ increasing? When is $f(x)$ decreasing? Justify your responses.

Inc: $(-3,-2)$ $f'(x) > 0$

Dec: $(-2,2)$ $(2,4)$ $f'(x) < 0$

b) Determine the x-coordinates where $f(x)$ has a relative maximum. Justify your response.

rel maximum @ $x = -2$

$f'(x)$ changes + to −

c) When is f concave up? When is f concave down? What are the x-coordinates of the points of inflection? Justify your responses.

CCU: $(0,2)$ $f'(x)$ increasing

CCD: $(-3,0)$ $(2,4)$ $f'(x)$ decreasing

P.O.I @ $x = 0, 2$

$f'(x)$ changes direction

d) Find an equation for the line tangent to the graph of f at the point $(0, 3)$.

$y - 3 = -2(x - 0)$