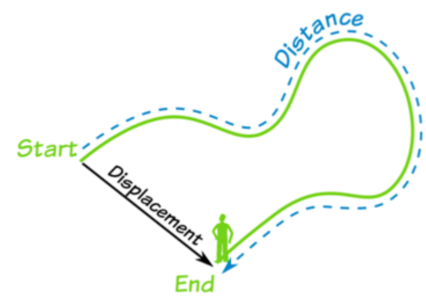
ABCALC Integrals as Net Change Notesheet **Name:** ____________________________

Recall that the definite integral gives us the net accumulation over an interval. For things that change, we can use the definite integral to model a myriad of real-world applications.

Distance versus Displacement

We have already seen how the position of an object can be found by finding the integral of the velocity function. The change in position is a displacement. To see the difference between distance and displacement, complete the following example.



Example 1 Consider the following statement

"Two steps forward and one step back"

What is the total distance traveled in this scenario? __3__

What is the total displacement in this scenario? __1__

To Find	Verbally	Mathematically
Displacement (Change in Position)	Integrate the rate of change over the interval	$\int_a^b v(t)dt$
Distance Traveled	Integrate the speed over the interval *Recall that speed is the absolute value of velocity	$\int_a^b speed\ dt = \int_a^b \lvert v(t)\rvert dt$
New Position	Old positon + change in position	$s(b) = s(a) + \int_a^b v(t)dt$

Example 2 Suppose the velocity of a particle moving along the x-axis is given by $v(t) = 6t^2 - 18t + 12$ when $0 \le t \le 2$.

$\to v(t) > 0 \to (0,1)$ $\quad \to v(t) < 0 \to (1,2)$ $\quad \to v(t) = 0 \to t = 1$ and 2

a) When is the particle moving to the right? When is the particle moving to the left? When is it stopped?

$v(t) = 6t^2 - 18t + 12 = 0 \qquad t = 1, 2$

$6(t^2 - 3t + 2) = 0 \qquad 6(t-2)(t-1) = 0$

$+$ $-$

$0 \quad 1 \quad 2$

b) Find the particle's displacement over the time interval.

$$\text{Displacement} = \int_0^2 (6t^2 - 18t + 12)\,dt = 2t^3 - 9t^2 + 12t\Big]_0^2 = 2(2)^3 - 9(2)^2 + 12(2) - [0]$$

$$= 16 - 36 + 24$$

$$= 40 - 36 = \boxed{4}$$

c) Find the particle's total distance traveled (calculator).

$$\text{Total Distance traveled} = \int_0^2 |v(t)|\,dt = 6$$

d) Setup an integral to find the particle's total distance traveled without using absolute value.

$$\int_0^1 v(t)\,dt - \int_1^2 v(t)\,dt = 6$$

Example 3 The tide removes sand from Sandy Point Beach at a rate modeled by the function R given by

$$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right)$$

A pumping station adds sand to the beach at a rate modeled by the function S, given by

volume/hr

$$S(t) = \frac{15t}{1+3t}$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand. (Calculator)

a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

$\int_0^6 R(t)\,dt = 31.816 \text{ yd}^3$

b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t.

$Y(t) = 2500 + \int_0^t S(t)\,dt - \int_0^t R(t)\,dt$

c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

$Y'(4) \rightarrow Y'(t) = S(t) - R(t) \rightarrow Y'(4) = S(4) - R(4)$

$= -1.909 \text{ yd}^3/\text{hr}$

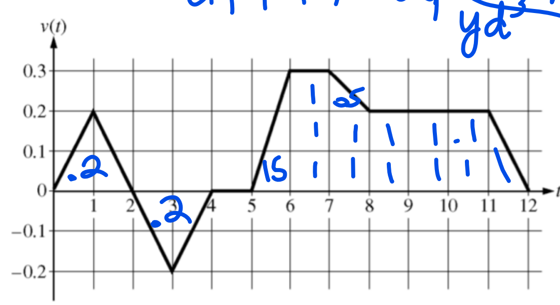
d) For $0 \le t \le 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answer.

$Y'(t) = 0 \quad S(t) - R(t) = 0 \quad Y(5.118) = 2{,}492.369 \text{ yd}^3$

$t = 5.118$

Example 4

Caren rides her bicycle along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes. During the time interval $0 \le t \le 12$ minutes, her velocity $v(t)$, in miles per minute, is modeled by the piecewise linear function whose graph is shown to the right. (Calculator)



a) Find the acceleration of Caren's bicycle at time $t = 7.5$ minutes. Indicate units of measure.

$a(t) = v'(t) \qquad a(7.5) = \frac{-.1}{1}$ miles/min/min

b) Using correct units, explain the meaning of $\int_0^{12} |v(t)|dt$ in terms of Caren's trip. Find $\int_0^{12} |v(t)|dt$

The distance traveled from $t=0$ to $t=12$. $= 1.8$ miles

c) Shortly after leaving home, Caren realizes that she left her calculus homework at home and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

$t = 2$ velocity changed from $+$ to $-$

d) Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is given by the function $w(t) = \frac{\pi}{15}\sin\left(\frac{\pi}{12}t\right)$, where $w(t)$ is in miles per minute for $0 \le t \le 12$ minutes. Who lives closer to the school, Caren or Larry? Show the work that leads to your answer.

$\int_0^{12} |w(t)|dt = 1.6$

Caren lives closer since she only lives 1.4 miles.