AB Calculus Integrals as Net Change Practice **Name:** ____________________

1. a) Integrating velocity gives displacement = change in position
(over a definite integral

b) Integrating the absolute value of velocity gives distance traveled.

c) New Position = old position + $\int_a^b v(t)\,dt$.
(at b) (will occur at a)

2. (Calculator) A particle starts at $x = 0$ and moves along the x-axis so that its velocity at time t is given by
(easy since initial position is 0)

$$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right)$$

a) Find the acceleration of the particle at time $t = 2$.

$a(t) = v'(t)$ $v'(2)$ or $a(2) = 1\ 588$

b) Find all times t in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.

$v(t) = 0$ $t = 2\ 507$ $v(t)$ changes $-$ to $+$

c) Find the total distance traveled by the particle from time $t = 0$ to time $t = 3$.

$\int_0^3 |v(t)|\,dt = 4\ 334$

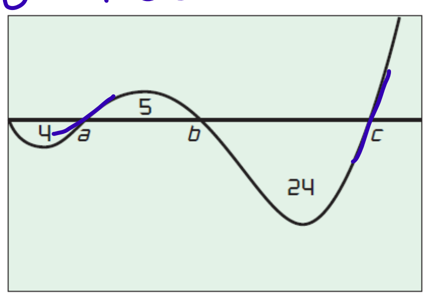
d) During the time interval $0 \le t \le 3$, what is the greatest distance between the particle and the origin? Show the work that leads to your answer. 3 candidates

dist=0 → $t = 0$ $t = 3$

$t = 2\ 507$ ⟶ $\int_0^{2\ 507} v(t)\,dt = -3\ 265$

$\begin{array}{r} 4\ 334 \\ -\ 3\ 265 \\ \hline 1\text{ something} \end{array}$

greatest distance $= 3\ 265$

3. 

A particle moves along the x-axis (units in cm). Its initial position at $t = 0$ seconds is $x(0) = 15$. The figure shows the graph of the particle's velocity $v(t)$. The numbers are the areas of the enclosed regions.

a) What is the particle's displacement between $t = 0$, and $t = c$?

$-4 + 5 - 24 = -23$ cm

b) What is the total distance traveled by the particle in the same time period as part a?

$4 + 5 + 24 = 33$ cm

c) Give the positions of the particle at times a, b, and c.

$x(a) = 11$ $x(b) = 16$ $x(c) = -8$

d) Approximately where does the particle achieve its greatest positive acceleration on the interval $[0, b]$?

at a

e) Approximately where does the particle achieve its greatest positive acceleration on the interval $[0, c]$?

at c

4. The rate at which people enter an amusement park on a given day is modeled by the function $E(t)$ defined by

$$E(t) = \frac{15600}{t^2 - 24t + 160}$$

The rate at which people leave the same amusement park is modeled by the function $L(t)$ defined by

$$L(t) = \frac{9890}{t^2 - 38t + 370}$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$. There are no people in the park.

a) How many people have entered the park by 5:00 pm ($t = 17$)? Round your answer to the nearest whole number.

$$\int_9^{17} E(t)\,dt = 6004 \text{ people}$$

b) The price of admission to the park is \$15 until 5:00 pm ($t = 17$). After 5:00 pm, the price of admission to the park is \$11. How much money is collected by the park on the given day? Round to the nearest dollar.

$$\$15 \times 6004 + \$11 \int_{17}^{23} E(t)\,dt = \$104{,}041$$

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c) Let $H(t) = \int_9^t \big(E(x) - L(x)\big)dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$ and explain the meaning of $H(17)$ and $H'(17)$ in the context of the park.

$H(17) = 3{,}725 \rightarrow$ # of people in park at 5 pm

$H'(17) = E(17) - L(17) = -380\ 281$ people/hr $\rightarrow$ at 5 pm people are leaving the park at a rate of 380.281 people/hr.

$H'(t) = E(t) - L(t)$

d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum? Show the work that leads to your answer.

$H'(t) = 0 = E(t) - L(t)$

at $t = 15\ 795$ $E(t)$ crosses $L(t)$