AB Calculus Evaluating Limits Algebraically Practice Name: Key

1. What are the four techniques we can use to prove the value of a limit?

direct substitution, factoring, simplifying (common denom), rationalizing (Conjugates)

Evaluate each limit without using a calculator.

2. $\lim_{x \to -2} x^3 + 6x^2 - 16$

$-8+24-16$

$= 0$

3. $\lim_{x \to 0} \pi^2$

$= \pi^2$ (Remember π is a constant)

4. $\lim_{x \to 4} \dfrac{x^2+9}{x^2-1} = \dfrac{4^2+9}{4^2-1} = \dfrac{25}{15} = \dfrac{5}{3}$

5. $\lim_{x \to 4} \dfrac{x^2-16}{x^2+x-20}$ $\quad \dfrac{16-16}{20-20} = \dfrac{0}{0}$

$\dfrac{(x+4)\cancel{(x-4)}}{(x+5)\cancel{(x-4)}} \rightarrow \dfrac{8}{9}$

6. $\lim_{x \to 1} \dfrac{1-x^2}{x^2+5x-6}$ $\quad \dfrac{1-1}{1+5-6} = \dfrac{0}{0}$

$\dfrac{(1-x)(1+x)}{(x+6)(x-1)} \rightarrow \dfrac{-\cancel{(x-1)}(1+x)}{(x+6)\cancel{(x-1)}} \rightarrow -\dfrac{2}{7}$

7. $\lim_{x \to 1} \dfrac{x^2+x-2}{x^2-4x+3}$ $\quad \dfrac{1+1-2}{1-4+3} = \dfrac{0}{0}$

$\dfrac{(x+2)\cancel{(x-1)}}{(x-3)\cancel{(x-1)}} \rightarrow \dfrac{3}{-2}$

8. $\lim_{x \to a} \dfrac{x^2-a^2}{x-a}$ $\quad \dfrac{a^2-a^2}{a-a} = \dfrac{0}{0}$

$\dfrac{\cancel{(x-a)}(x+a)}{\cancel{x-a}} \rightarrow a+a = 2a$

9. $\lim_{x \to 3} \dfrac{x^3-27}{x-3}$ $\quad \dfrac{3^3-27}{3-3} = \dfrac{0}{0}$

$\dfrac{\cancel{(x-3)}(x^2+3x+9)}{\cancel{x-3}} \rightarrow 9+9+9 = 27$

10. $\lim_{x \to 1} \dfrac{x^3-3x^2+2x}{x-1}$ $\quad \dfrac{1-3+2}{1-1} = \dfrac{0}{0}$

$x(x^2-3x+2)$

$\dfrac{x(x-2)\cancel{(x-1)}}{\cancel{x-1}} \rightarrow 1(-1) = -1$

11. $\lim_{x \to 6} 10$

10

12. $\lim_{x \to 11} \dfrac{\sqrt{x-2}-3}{x-11} \cdot \dfrac{\sqrt{x-2}+3}{\sqrt{x-2}+3}$

$\dfrac{x-2-9}{(x-11)(\sqrt{x-2}+3)} = \dfrac{\cancel{x-11}}{\cancel{(x-11)}(\sqrt{x-2}+3)} \rightarrow \dfrac{1}{\sqrt{9}+3} = \dfrac{1}{6}$

13. $\lim_{x \to 10^-} \dfrac{|x-10|}{x-10}$

$f(x) = \begin{cases} 1, & x \geq 10 \\ -1, & x < 10 \end{cases}$ left

$= -1$

14. $\lim_{x \to 2} \sqrt{x-2}$

$\sqrt{2-2} = \sqrt{0} = 0$

but $\sqrt{x-2}$ starts @ $x = 2$, so since it's undefined to the left of 2, the limit DNE

15. $\lim_{x \to 1} f(x)$ if $f(x) = \begin{cases} \dfrac{1}{x+2}, & x < 1 \\ 1-2x, & x \geq 1 \end{cases}$

$\dfrac{1}{1+2} = \dfrac{1}{3}$

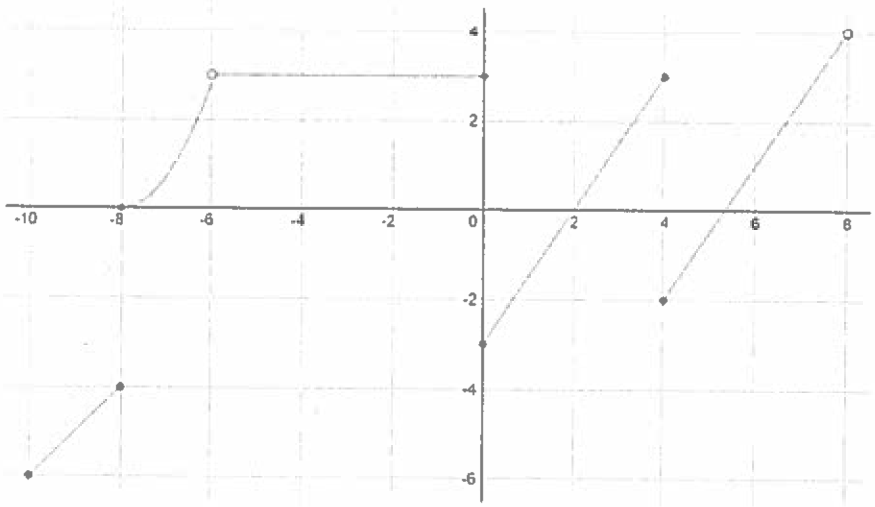
$1-2(1) = -1$

LHL $\neq$ RHL

DNE

Limits and Graphs Practice Name: ____________________

Find the following limits.



1. $\lim_{x\to 0^-} f(x) =$ 3

2. $\lim_{x\to 0^+} f(x) =$ −3

3. $\lim_{x\to 8^+} f(x) =$ DNE

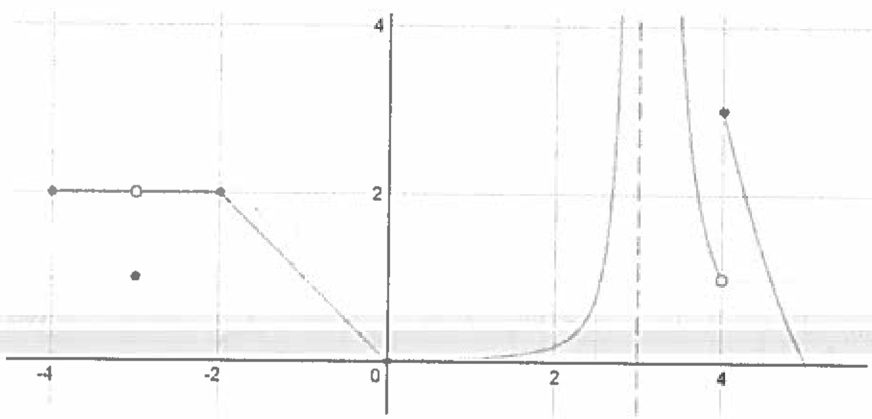
4. $\lim_{x\to -8} f(x) =$ DNE

4. $\lim_{x\to 8^-} f(x) =$ 4

6. $\lim_{x\to 0} f(x) =$ DNE

5. $\lim_{x\to 4^+} f(x) =$ −2

8. $\lim_{x\to 4} f(x) =$ DNE



9. $\lim_{x\to -3} f(x) =$ 2

10. $\lim_{x\to 3} f(x) =$ ∞

11. $\lim_{x\to 0} f(x) =$ 0

12. $f(-3) =$ 1

13. $\lim_{x\to 4^-} f(x) =$ 1

14. $f(3) =$ undefined

15. $\lim_{x\to 4^+} f(x) =$ 3

16. $f(4) =$ 3