

1. How do you find horizontal asymptotes? *simplify 1st!* if degrees = if degree of den > deg of num. $y=0$ then $y = \frac{a}{b}$
 2. How do you find vertical asymptotes? *zeros of den. after simplifying* \downarrow Coeff.

3. For each of the following, find $\lim_{x \rightarrow \infty} f(x)$, $\lim_{x \rightarrow -\infty} f(x)$ and identify all horizontal asymptotes.

a) $f(x) = \frac{x-2}{2x^2+3x-5}$ *$\frac{x-2}{(2x+5)(x-1)}$* $\lim_{x \rightarrow \infty} f(x) = 0$ $\lim_{x \rightarrow -\infty} f(x) = 0$ H.A. $y=0$

b) $f(x) = \frac{4x^3-2x+1}{x^2-2x+1}$ *EBM $y=4x$* $\lim_{x \rightarrow \infty} f(x) = \infty$ $\lim_{x \rightarrow -\infty} f(x) = -\infty$ No H.A.

c) $f(x) = \frac{3x^2-x+5}{x^2-4}$ *EBM $y=3$* $\lim_{x \rightarrow \infty} f(x) = 3$ $\lim_{x \rightarrow -\infty} f(x) = 3$ H.A. $y=3$

d) $f(x) = \frac{|x|}{x}$ *$f(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$* $\lim_{x \rightarrow \infty} f(x) = 1$ $\lim_{x \rightarrow -\infty} f(x) = -1$ no H.A.

4. For each of the following, identify all vertical asymptotes and find $\lim_{x \rightarrow a^-} f(x)$, $\lim_{x \rightarrow a^+} f(x)$, and $\lim_{x \rightarrow a} f(x)$, where a is the x-value of the asymptote.

a) $f(x) = \frac{1}{x-3}$ *V.A. $x=3$* $\lim_{x \rightarrow 3^-} f(x) = -\infty$ $\lim_{x \rightarrow 3^+} f(x) = \infty$ $\lim_{x \rightarrow 3} f(x)$ DNE

b) $f(x) = \frac{1}{x^2-4x+4}$ *$\frac{1}{(x-2)(x-2)}$* $\lim_{x \rightarrow 2^-} f(x) = \infty$ $\lim_{x \rightarrow 2^+} f(x) = \infty$ $\lim_{x \rightarrow 2} f(x) = \infty$
 V.A. $x=2$

c) $f(x) = \frac{x^2-3x-4}{x^2-16}$ *$\frac{(x-4)(x+1)}{(x-4)(x+4)}$* *V.A. $x=-4$* $\lim_{x \rightarrow -4^-} f(x) = \infty$ $\lim_{x \rightarrow -4^+} f(x) = -\infty$ $\lim_{x \rightarrow -4} f(x)$ DNE

5. For each of the following piecewise functions, find the limit as $x \rightarrow \infty$, $x \rightarrow -\infty$, $x \rightarrow 0^-$, and $x \rightarrow 0^+$

a) $g(x) = \begin{cases} \frac{1}{x} & x < 0 \\ \frac{2x-3}{x+1} & x \geq 0 \end{cases}$

$\lim_{x \rightarrow \infty} g(x) = 2$
 $\lim_{x \rightarrow -\infty} g(x) = 0$
 $\lim_{x \rightarrow 0^-} g(x) = -\infty$
 $\lim_{x \rightarrow 0^+} g(x) = -3$

b) $g(x) = \begin{cases} \frac{3x}{x+1} & x \leq 0 \\ \frac{1}{x^2} & x > 0 \end{cases}$

$\lim_{x \rightarrow \infty} g(x) = 0$
 $\lim_{x \rightarrow -\infty} g(x) = 3$
 $\lim_{x \rightarrow 0^-} g(x) = 0$
 $\lim_{x \rightarrow 0^+} g(x) = \infty$

6. Sketch a function that satisfies the stated conditions. Include any asymptotes.

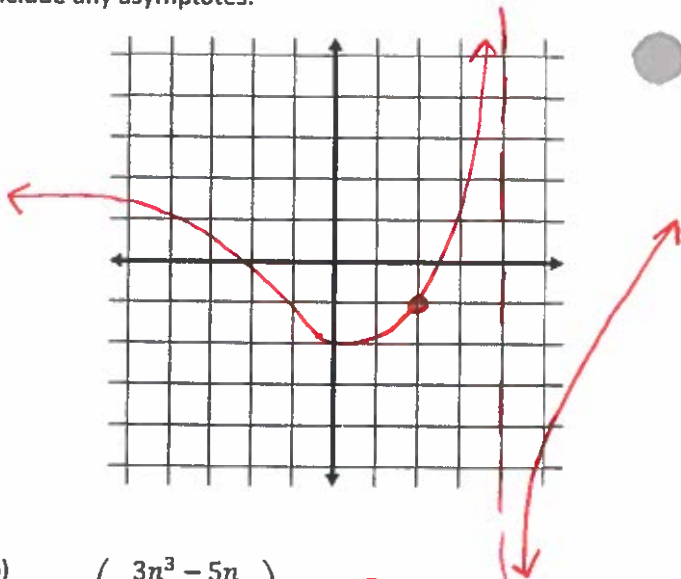
$$\lim_{x \rightarrow 2} f(x) = -1 \quad \checkmark$$

$$\lim_{x \rightarrow 4^-} f(x) = \infty \quad \checkmark$$

$$\lim_{x \rightarrow -\infty} f(x) = 2 \quad \checkmark$$

$$\lim_{x \rightarrow 4^+} f(x) = -\infty \quad \checkmark$$

$$\lim_{x \rightarrow \infty} f(x) = \infty \quad \checkmark$$



7. Evaluate the following limits without a calculator.

a) $\lim_{x \rightarrow \infty} \left(\frac{2}{x} + 1\right) \left(\frac{5x^2 - 1}{x^2}\right)$

$$\frac{2}{\infty} + 1 \quad \checkmark$$

$$(0+1)(5) = 5$$

b) $\lim_{n \rightarrow \infty} \left(\frac{3n^3 - 5n}{n^3 - 2n^2 + 1}\right) = 3$

c) $\lim_{x \rightarrow \infty} e^{-x} \cos x$

$$e^{-\infty}$$

$$\frac{1}{e^x} \cdot \cos x \rightarrow \frac{\cos x}{e^x} = 0$$

↖ -1 ↔ 1
↘ Big ∞

d) $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 + 10x + 21}$

$$\frac{(x+3)(x-3)}{(x+7)(x+3)} = \frac{x-3}{x+7}$$

$$= \frac{0}{10} = 0$$

8. Use the graph of $f(x)$ below to evaluate the following limits.

a) $f(-1) = -1$

b) $\lim_{x \rightarrow -\infty} f(x) = 4$

c) $\lim_{x \rightarrow \infty} f(x) = 0$

d) $\lim_{x \rightarrow 3} f(x) = -\infty$

e) $\lim_{x \rightarrow 2} f(x) = -1$

f) $\lim_{x \rightarrow -1} f(x) = 2$

