$L(x) \approx y = m(x-x_1)+y_1$

AP Calculus Linearization and Differentials **Name:** ______________________________

1. If $f(x) = x^3 - 2x + 3$, $y - y_1 = m(x-x_1)$

   a) Find the linearization $L(x)$ of $f(x)$ centered at $x = 2$.

   $f'(x) = 3x^2 - 2$ $y - 7 = 10(x-2)$ $\boxed{L(x) \approx 10(x-2)+7}$

   $f(2) = 7$ $f'(2) = 10$

   b) Use $L(x)$ to approximate $f(2.1)$

   $L(2.1) \approx 10(2.1-2)+7 = 8$

   c) Determine if the approximation is an underestimate or an overestimate.

   $f''(x) = 6x$ since $f''(2) > 0$ (therefore concave up)

   d) Use a calculator to determine the accuracy of the approximation you found in part b.

   $|8.061 - 8| = .061 \rightarrow$ error

2. If $f(x) = \sqrt{x^2+9}$, $f(-4) = 5$ $y - 5 = \frac{-4}{5}(x+4)$

   a) Find the linearization $L(x)$ of $f(x)$ centered at $x = -4$.

   $f'(x) = \frac{1}{2}(x^2+9)^{-\frac{1}{2}}(2x)$ $\boxed{L(x) \approx -\frac{4}{5}(x+4)+5}$

   $f'(-4) = \frac{1}{2}((-4)^2+9)^{-\frac{1}{2}}(2(-4)) = \frac{-4}{5}$

   b) Use $L(x)$ to approximate $f(-3.9)$

   $L(-3.9) \approx \frac{-4}{5}(-3.9+4)+5 = \frac{-4}{50}+5$ or $-.08+5 = 4.92$

   $.1 = \frac{1}{10}$

   c) Use a calculator to determine the accuracy of the approximation you found in part b.

   $4.9203658 - 4.92 = 0.00365\ldots$ = error

3. Approximate the value of $\sqrt{101}$ by using a linearization of a nearby number.

   $f(x) = \sqrt{x}$ (center at $x = 100$)

   $f'(x) = \frac{1}{2\sqrt{x}}$ $y - 10 = \frac{1}{20}(x-100)$

   $L(x) \approx \frac{1}{20}(x-100)+10$

   $L(101) \approx \frac{1}{20}(101-100)+10$

   $f(100) = 10$ $f'(100) = \frac{1}{2\sqrt{100}} = \frac{1}{2 \cdot 10} = \frac{1}{20}$ ≈ 10.05

4. For each of the following, find dy and evaluate dy for the given value of x and dx.

a) $y = x^3 - 3x$, $x = 2$, $dx = 0.05$

$$\frac{dy}{dx} = 3x^2 - 3 \rightarrow dy = (3x^2 - 3)dx$$

$$dy(x=2, dx=.05) = (3(2)^2 - 3).05 = .45$$

b) $y = x^2 \ln x$, $x = 1$, $dx = 0.01$

$$\frac{dy}{dx} = 2x\ln x + x^2 \frac{1}{x}$$

$$\frac{dy}{dx} = 2x\ln x + x \rightarrow dy = (2x\ln x + x)dx$$

$$dy(x=1, dx=.01) = (2 \cdot 1 \ln 1 + 1).01$$

$$\boxed{= .01}$$

5. Use differentials to estimate the change in the volume of a sphere $\left(V = \frac{4}{3}\pi r^3\right)$ when the radius changes from 10 cm to 10.05 cm.

dV $\quad$ $dr = .05$

$$\frac{dV}{dr} = \frac{4}{3}\pi(3r^2)$$

$$dV = 4\pi r^2 dr$$

$$dV(r=10, dr=.05) = 4\pi(10)^2(.05) = 20\pi$$

6. Evaluate the following limits.

a) $\lim\limits_{x\to 0^+} x \ln x$

$$\lim_{x\to 0^+} \frac{\ln x}{1/x} \rightarrow \frac{-\infty}{\infty} \rightarrow \text{L'Hospitals}$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} \rightarrow \frac{1}{x} \cdot \frac{-x^2}{1} \rightarrow -x = 0$$

- Try these (back of notes) while you wait
- Also ask each ??'s about RR HW
- I'll be down around 9

b) $\lim\limits_{x\to \frac{\pi}{2}^+} \frac{1 + \sec x}{\tan x} \rightarrow \frac{\infty}{\infty} \rightarrow \text{L'H}$

$$\lim_{x\to \frac{\pi}{2}^+} \frac{\sec x \tan x}{\sec^2 x} \rightarrow \frac{\tan x}{\sec x} \rightarrow \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} \rightarrow \sin x = 1$$

c) $\lim\limits_{x\to 0} (e^x + x)^{\frac{1}{x}}$