AP Calculus Linearization and Differentials Homework Name: ______________________

1. If $f(x) = x^3 - 2x + 3$,

   a) Find the linearization $L(x)$ of $f(x)$ centered at $x = 2$.

$x=2$ $\quad f'(x) = 3x^2 - 2$ $\quad$ TL $y - 7 = 10(x-2)$

$f(2) = 7$ $\quad f'(2) = 10$ $\quad L(x) \approx y = 10(x-2) + 7$

   b) Use $L(x)$ to approximate $f(2.1)$

$f(2.1) \approx L(2.1) = 10(2.1 - 2) + 7$

$= 10(.1) + 7$

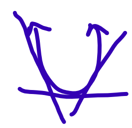
$= 1 + 7 = 8$

   c) Determine if the approximation is an underestimate or an overestimate.

$L(2.1) = 8$ $\quad$ we underestimated $\quad$ 2nd derivative $\rightarrow f''(x) = 6x$

at $f''(2) = 12 > 0$, ccup

$f(2.1) = 8.061$ $\quad$ w/o using a calc, How do I know?



   d) Use a calculator to determine the accuracy of the approximation you found in part b.

$|8.061 - 8| = .061 < .1$

error

2. If $f(x) = \sqrt{x^2 + 9}$,

   a) Find the linearization $L(x)$ of $f(x)$ centered at $x = -4$.

$f'(x) = \frac{1}{2} \frac{2x}{\sqrt{x^2+9}} = \frac{x}{\sqrt{x^2+9}}$ $\quad f(-4) = \sqrt{16+9} = 5$

$y - 5 = -\frac{4}{5}(x+4)$

$f'(-4) = \frac{-4}{\sqrt{16+9}} = -\frac{4}{5}$ $\quad \boxed{L(x) \approx y = -\frac{4}{5}(x+4) + 5}$

   b) Use $L(x)$ to approximate $f(-3.9)$

$f(-3.9) \approx L(-3.9) = -\frac{4}{5}(-3.9 + 4) + 5$ $\quad$ or $-.08 + 5 = 4.92$

$= -\frac{4}{5} \cdot \frac{1}{10} + 5 = -\frac{2}{25} + \frac{125}{25} = \frac{123}{25}$

   c) Use a calculator to determine the accuracy of the approximation you found in part b.

$f(-3.9) = 4.92036584$

$|f(-3.9) - L(-3.9)| = 3.6584 \times 10^{-4} < \frac{1}{1000}$

3. Approximate the value of $\sqrt{101}$ by using a linearization of a nearby number.

use $x = 100$

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$ $\quad y - 10 = \frac{1}{20}(x - 100)$

$L(x) = \frac{1}{20}(x - 100) + 10$

$f(100) = \sqrt{100} = 10$ $\quad L(101) = \frac{1}{20}(101 - 100) + 10$

$f'(100) = \frac{1}{2\sqrt{100}} = \frac{1}{20}$ $\quad = \frac{1}{20} + 10$ $\quad L(101) = 10.05$

$\sqrt{101} \approx 10.05$

4. For each of the following, find dy and evaluate dy for the given value of x and dx.

a) $y = x^3 - 3x,\ x = 2,\ dx = 0.05$

$$\frac{dy}{dx} = 3x^2 - 3$$

$$dy = (3x^2 - 3)\ dx$$

$$dy\Big|_{x=2,\ dx=05} = (3\,2^2 - 3)\ \ 05$$

$$= 9\ \ 05$$

$$\boxed{dy\Big|_{2,05} = 45}$$

b) $y = x^2 \ln x,\ x = 1,\ dx = 0.01$

$$\frac{dy}{\cancel{dx}} = \left(2x \ln x + x^2\ \frac{1}{x}\right) dx$$

$$\frac{dy}{dx}\Big|_{x=1,\ dx=01} = \left(2(1)\ \ln 1 + 1^2\ \frac{1}{1}\right)\ 01 = \boxed{01 = dy}$$

5. Use differentials to estimate the change in the volume of a sphere $\left(V = \frac{4}{3}\pi r^3\right)$ when the radius changes from 10 cm to 10.05 cm

$dr = 10.05 - 10 = 05$

$$\frac{dV}{dr} = 4\pi r^2$$

$$dV = 4\pi r^2 dr$$

$$dV = 4\pi(10)^2(05) = \frac{4\pi\ 100\ \ 5}{100} = \boxed{20\pi\ cm^3}$$

6. Evaluate the following limits.

a) $\lim\limits_{x \to 0^+} x \ln x$

$$\lim_{x \to 0^+} \frac{\ln x}{1/x} \to \frac{-\infty}{\infty} \to \text{L'Hospital's} = \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} \to \frac{-x^2}{x} \to -x \to \boxed{0}$$

b) $\lim\limits_{x \to \frac{\pi}{2}^+} \dfrac{1 + \sec x}{\tan x}$

$$\to \frac{\frac{\cos x}{\cos x} + \frac{1}{\cos x}}{\frac{\sin x}{\cos x}}$$

$$-\frac{\infty}{\infty} \to \text{L'H} \to \frac{\sec x \tan x}{\sec^2 x} \to \frac{\tan x}{\sec x} \xrightarrow{\text{UH}} \frac{\sec^2 x}{\sec x \tan x}$$

$$\to \frac{\sec x}{\tan x} \to \frac{\frac{1}{\cos x}}{\frac{\sin x}{\cos x}} = \frac{1}{\sin x}$$

$$\lim_{x \to \frac{\pi}{2}^+} \frac{\cos x + 1}{\sin x} = \frac{0 + 1}{1} = 1$$

c) $\lim\limits_{x \to 0} (e^x + x)^{\frac{1}{x}}$

take ln of this → Final step is to undo ln by raising e^

$$\boxed{e^2}$$

$$\lim_{x \to 0} \frac{1}{x} \ln(e^x + x)$$

$$\lim_{x \to 0} \frac{\ln(e^x + x)}{x} \to \frac{0}{0} \to \text{L'Hospitals}$$

$$= \lim_{x \to 0} \frac{\frac{1}{e^x + x}(e^x + 1)}{1} = \frac{1}{1}(2) = 2$$