AB Calculus Mean Value Theorem Day 2 Name: ____________________

1. For each of the following, find the open intervals where the function is increasing and decreasing and find all relative extrema.

Inc $(-\infty,0)$ $(2,\infty)$
Dec $(0,1)$ $(1,2)$

a) $y = \dfrac{x^2}{2x-2}$

rel max at $x=0$
rel min at $x=2$

$\frac{dy}{dx} = \frac{(2x-2)(2x)-(x^2)(2)}{(2x-2)^2}$

$\frac{dy}{dx} = \frac{4x^2-4x-2x^2}{(2x-2)^2} = \frac{2x^2-4x}{(2x-2)^2} = \frac{2x(x-2)}{(2x-2)^2}$

cp $x=0,2$
(point of interest $x=1$)

Sign chart: + | − | − | + at 0, 1, 2

b) $y = -x^3 + 2x^2 + 4$

Inc $(0,\frac{4}{3})$
Dec $(-\infty,0)(\frac{4}{3},\infty)$

$\frac{dy}{dx} = -3x^2+4x$
$0 = x(-3x+4)$
cp $x=0, \frac{4}{3}$

rel min at $x=0$
rel max at $x=4/3$

Sign chart: − | + | − at 0, 4/3

2. For each of the following, find the absolute extrema over the given interval

a) $y = -2x^2 - 4x + 4$ over $[-1, 1]$

$\frac{dy}{dx} = -4x-4$
$0=-4x-4$
$x=-1$

abs max $= 6$ at $x=-1$
abs min $= -2$ at $x=1$

x	y
−1	$-2(-1)^2-4(-1)+4 = 6$
1	$-2(1)^2-4(1)+4 = -2$

b) $y = 2\sin^2 x$ over $\left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$

$y = 2(\sin x)^2$
$y' = 4(\sin x)^1 \cos x$
$0 = 4\sin x \cos x$
$\sin x = 0$ at $x=0$
$\cos x = 0$ at $x=\frac{\pi}{2}$

x	y
$-\pi/6$	$1/2$
0	0
$\pi/3$	$3/2$

abs min $= 0$ at $x=0$
abs max $= 3/2$ at $x=\pi/3$

3. For each of the following, show that the function f satisfies the hypothesis of the Mean Value Theorem on the given interval $[a,b]$. If it does, find each value of c in (a,b) guaranteed by the theorem.

a) $f(x) = \dfrac{x^2-1}{4x}$ over $[-1,1]$

No MVT (why?)
Not continuous at $x=0$
and 0 is between $[-1,1]$

b) $y = x^3 + 10x^2 + 32x + 33$ over $[-4,-2]$

Polynomials are all cont & diff

$y' = 3x^2+20x+32$

$y(-2) = (-2)^3+10(-2)^2+32(-2)+33 = 1$
$y(-4) = (-4)^3+10(-4)^2+32(-4)+33 = 1$

$\frac{y(-2)-y(-4)}{-2--4} = \frac{1-1}{2} = 0$

$x = -\frac{8}{3}$

$3x^2+20x+32=0$
$(3x+8)(x+4)=0$
$x = -\frac{8}{3}, -4 \rightarrow$ endpts don't get justified by MVT

4. Find $\frac{dy}{dx}$ for each of the following

a) $y = \sec\left(e^{5x^4}\right)$

b) $y = \sin\left(2^{x^5}\right)$

c) $y = \csc(\ln(3x^2))$

d) $y = \ln(2 + e^{2x^5})$

e) $2x - 3xy^2 = 4$

f) $\csc(y^2) = 5x + 4$

g) $y = (\sin x)^{3x}$

h) $2x^2 + 3 = e^{2y^3}$

i) $f(x) = \ln x \cdot \cot^{-1} x$

j) $2x + 1 = \ln(2y^2)$

5. If $2 = 2(x^2 + y)^3 + 2y$, find $\frac{dy}{dx}$ at $(-1, 0)$.