

1. For each of the following, find the open intervals where the function is increasing and decreasing and find all relative extrema.

a)  $y = \frac{x^2}{2x-2}$

Inc  $(-\infty, 0) (2, \infty)$   
Dec  $(0, 1) (1, 2)$

$$\frac{dy}{dx} = \frac{(2x-2)(2x) - (x^2)(2)}{(2x-2)^2}$$

rel max at  $x=0$   
rel min at  $x=2$

$$\frac{dy}{dx} = \frac{4x^2 - 4x - 2x^2}{(2x-2)^2} = \frac{2x^2 - 4x}{(2x-2)^2} = \frac{2x(x-2)}{(2x-2)^2}$$

cp  $x=0, 2$   
(point of interest  $x=1$ )



b)  $y = -x^3 + 2x^2 + 4$

Inc  $(0, \frac{4}{3})$

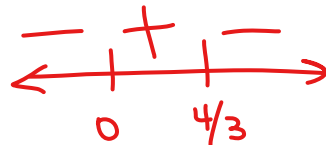
Dec  $(-\infty, 0) (\frac{4}{3}, \infty)$

$$\frac{dy}{dx} = -3x^2 + 4x$$

$$0 = x(-3x+4)$$

cp  $x=0, \frac{4}{3}$

rel min at  $x=0$   
rel max at  $x=\frac{4}{3}$



2. For each of the following, find the absolute extrema over the given interval

a)  $y = -2x^2 - 4x + 4$  over  $[-1, 1]$

b)  $y = 2 \sin^2 x$  over  $[-\frac{\pi}{6}, \frac{\pi}{3}]$

$$\frac{dy}{dx} = -4x - 4$$

$$0 = -4x - 4$$

$$x = -1$$

abs max = 6  
at  $x = -1$

abs min = -2  
at  $x = 1$

x	y
-1	$-2(-1)^2 - 4(-1) + 4 = 6$
1	$-2(1)^2 - 4(1) + 4 = -2$

$$y = 2(\sin x)^2$$

$$y' = 4(\sin x) \cos x$$

$$0 = 4 \sin x \cos x$$

$$\sin x = 0 \text{ at } x = 0$$

$$\cos x = 0 \text{ at } x = \frac{\pi}{2}$$

x	y
$-\frac{\pi}{6}$	$\frac{1}{2}$
0	0
$\frac{\pi}{3}$	$\frac{3}{2}$

abs min = 0  
at  $x = 0$

abs max =  $\frac{3}{2}$   
at  $x = \frac{\pi}{3}$

3. For each of the following, show that the function  $f$  satisfies the hypothesis of the Mean Value Theorem on the given interval  $[a, b]$ . If it does, find each value of  $c$  in  $(a, b)$  guaranteed by the theorem.

a)  $f(x) = \frac{x^2 - 1}{4x}$  over  $[-1, 1]$

No MVT (why?)

Not continuous at  $x=0$   
and 0 is between  $[-1, 1]$

b)  $y = x^3 + 10x^2 + 32x + 33$  over  $[-4, -2]$

Polynomials are all cont & diff

$$y' = 3x^2 + 20x + 32$$

$$y(-2) = (-2)^3 + 10(-2)^2 + 32(-2) + 33 = 1$$

$$y(-4) = (-4)^3 + 10(-4)^2 + 32(-4) + 33 = 1$$

$$\frac{y(-2) - y(-4)}{-2 - (-4)} = \frac{1 - 1}{2} = 0$$

$$x = -\frac{8}{3}$$

$$3x^2 + 20x + 32 = 0$$

$$(3x + 8)(x + 4) = 0$$

$x = -\frac{8}{3}, -4 \rightarrow$  endpoints don't get justified by MVT

4. Find  $\frac{dy}{dx}$  for each of the following

a)  $y = \sec(e^{5x^4})$

b)  $y = \sin(2^{x^5})$

c)  $y = \csc(\ln(3x^2))$

d)  $y = \ln(2 + e^{2x^5})$

e)  $2x - 3xy^2 = 4$

f)  $\csc(y^2) = 5x + 4$

g)  $y = (\sin x)^{3x}$

h)  $2x^2 + 3 = e^{2y^3}$

i)  $f(x) = \ln x \cdot \cot^{-1} x$

j)  $2x + 1 = \ln(2y^2)$

5. If  $2 = 2(x^2 + y)^3 + 2y$ , find  $\frac{dy}{dx}$  at  $(-1, 0)$ .