

1. State the Mean Value Theorem (MVT) in 2 ways:

a) In words: cont $[a, b]$ diff (a, b) A.R $OC = \frac{1}{R} OC$ or slope = derivative

b) Mathematically:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

2. For each of the following, (a) state whether or not the function satisfies the hypothesis of the MVT over the given interval, and (b) if it does, find that value of c that the MVT guarantees.

cont $[1, 6]$
diff $(1, 6)$

a) $f(x) = -2x^2 + 14x - 12$ over $[1, 6]$

$f'(x) = -4x + 14$
 $f(6) = -2(36) + 14(6) - 12 = -72 + 84 - 12 = 0$
 $f(1) = -2(1) + 14 - 12 = 12 - 12 = 0$

b) $h(x) = x^{1/3}$ over $[-1, 1]$

$\frac{0 - 0}{1 - (-1)} = 0$

NO MVT SINCE $h(x)$ is NOT diff on $(-1, 1)$

$-4x + 14 = 0$
 $-4x = -14$
 $x = \frac{-14}{-4} = \frac{7}{2}$ → value that's guaranteed by MVT

3. Suppose $f(x)$ is a differentiable function on the interval $[-7, 1]$ such that $f(-7) = 4$ and $f(1) = -1$.

a) Explain why f must have at least one value in the interval $(-7, 1)$, where the function equals 2. → y-value

IVT b/c $-1 < 2 < 4$ there is a c such $-7 < c < 1$ that $f(c) = 2$

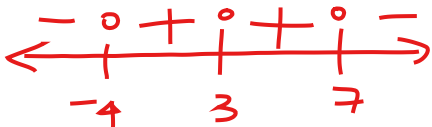
b) Explain why there must be at least one point on the interval $(-7, 1)$ where the derivative is $-\frac{5}{8}$.

MVT $\frac{-1 - 4}{1 - (-7)} = -\frac{5}{8}$

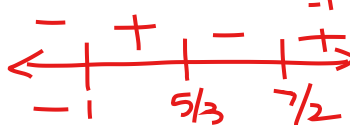
therefore there is a $c < 1$ such that $f'(c) = -\frac{5}{8}$

4. Make a sign chart for the following functions.

a) $f'(x) = (x - 3)^2(x + 4)(7 - x)$



b) $g'(x) = \frac{5(2x - 7)}{(x + 1)(3x - 5)}$



5. For each function, find the critical numbers of f and where the function is increasing or decreasing and find all relative extrema.

a) $h(x) = \frac{2}{x}$

b) $f(x) = x^3 - 6x^2 + 15$

c) $h(x) = \frac{-x}{x^2 + 4}$

d) $g(x) = x^2 - x - \ln x$