AB Calculus: Mean Value Theorem Function Practice **Name:** ______________________________

1. State the Mean Value Theorem (MVT) in 2 ways:

a) In words:

b) Mathematically:

2. For each of the following, (a) state whether or not the function satisfies the hypothesis of the MVT over the given interval, and (b) if it does, find that value of c that the MVT guarantees.

a) $f(x) = -2x^2 + 14x - 12$ over $[1, 6]$ b) $h(x) = x^{1/3}$ over $[-1, 1]$

3. Suppose $f(x)$ is a differentiable function on the interval [-7, 1] such that $f(-7) = 4$ and $f(1) = -1$.

a) Explain why f must have at least one value in the interval (-7, 1), where the function equals 2.

IVT $-1 < 2 < 4$ so there is an $f(c) = 2$ $-7 < c < 1$

b) Explain why there must be at least one point on the interval (-7, 1) where the derivative is $-\frac{5}{8}$.

MVT $\frac{f(b) - f(a)}{b - a} = f'(c)$ $\frac{-5}{8} = \frac{-1 - 4}{1 - (-7)}$

4. Make a sign chart for the following functions.

a) $f'(x) = (x - 3)^2(x + 4)(7 - x)$

$-$ | $+$ | $+$ | $-$ $f'(x)$

-4 3 7

b) $g'(x) = \frac{5(2x - 7)}{(x + 1)(3x - 5)}$ $7/2$ $5/3$ -1

$-$ | $+$ | $-$ | $+$ $f'(x)$

-1 $5/3$ $7/2$

5. For each function, find the critical numbers of f and where the function is increasing or decreasing and find all relative extrema.

a) $h(x) = \frac{2}{x}$ $\rightarrow h(x) = 2x^{-1}$ so $h'(x) = -2x^{-2}$ or $\frac{-2}{x^2} \rightarrow x = 0$

$-$ | und $-$ $h'(x)$
0
decreasing $(-\infty, 0)$ $(0, \infty)$
NO Rel extrema

b) $f(x) = x^3 - 6x^2 + 15$ $f'(x) = 3x^2 - 12x$
$0 = 3x(x - 4)$ CP $x = 0, 4$

$+$ | $-$ | $+$
0 4

Inc $(-\infty, 0)$ $(4, \infty)$
Dec: $(0, 4)$
rel max at $x = 0$
rel min at $x = 4$

c) $h(x) = \frac{-x}{x^2 + 4}$ $h'(x) = \frac{(x^2+4)(-1) - (-x)(2x)}{(x^2+4)^2} = \frac{-x^2 - 4 + 2x^2}{(x^2+4)^2} = \frac{x^2 - 4}{(x^2+4)^2}$

$\frac{(x+2)(x-2)}{(x^2+4)^2} \rightarrow$ $+$ | $-$ | $+$
-2 2

Inc $(-\infty, -2)$ $(2, \infty)$
Dec: $(-2, 2)$ rel max at $x = -2$
rel min at $x = 2$

d) $g(x) = x^2 - x - \ln x$

$g'(x) = 2x - 1 - \frac{1}{x}$

$\boxed{\begin{array}{l}\text{Inc } (1, \infty) \\ \text{Dec } (0, 1) \\ \text{rel min at } x = 1\end{array}}$

domain starts

$-$ | $+$
0 1

$0 = \frac{2x^2 - x - 1}{x} \rightarrow \frac{(2x+1)(x-1)}{x}$ only CP $x = 1$
(since $x \neq 0, -\frac{1}{2}$ in $\ln x$)