AB Calculus Rates of Change and Tangent Lines Practice **Name:** ______________________________

1. What is the difference quotient?

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

2. How do you find the slope of a curve at a point (a.k.a. slope of the tangent line to a curve) at $x = a$?

blue stuff makes red slope INSTANTANEOUS

3. What is a normal line?

perpendicular (slopes are opposite, reciprocal)

4. What is the difference between average rate of change and instantaneous rate of change?

→ algebraic slope between 2 pts → slope a secant line

→ derivative → slope of a tangent line

5. Find the average rate of change of each function over the indicated interval.

a) $h(x) = 2 + \sin x$ over $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$h\left(\frac{\pi}{2}\right) = 2 + \sin\left(\frac{\pi}{2}\right) = 2 + 1 = 3$

$h\left(-\frac{\pi}{2}\right) = 2 + \sin\left(-\frac{\pi}{2}\right) = 2 + (-1) = 1$

$\frac{3 - 1}{\frac{\pi}{2} - -\frac{\pi}{2}}$

$\frac{2}{\pi}$

b) $f(x) = x^2 - x$ over $[2, 5]$

$f(2) = 2^2 - 2 = 4 - 2 = 2$

$f(5) = 5^2 - 5 = 25 - 5 = 20$

$\frac{20 - 2}{5 - 2} = \frac{18}{3} = 6$

6. Let $f(x) = x^3$

a) Write and simplify an expression for $f(x + h)$.

$f(x+h) = x^3 + 3x^2h + 3xh^2 + h^3$

b) Find the slope of $f(x)$ at $x = x$.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2\cancel{h} + 3xh^{\cancel{2}} + h^{\cancel{3}2} - \cancel{x^3}}{\cancel{h}} \rightarrow \lim_{h \to 0} 3x^2 + \cancel{3xh} + \cancel{h^2}$$

$\boxed{f'(x) = 3x^2}$

c) When does the slope equal 12?

$3x^2 = 12 \qquad x^2 = 4 \qquad x = \pm 2$

d) Write the equation of the tangent line to the curve at $x = 4$.

$y - y_1 = m(x - x_1)$

$x_1 = 4$

$y_1 = 64$

$m_{\text{at } x=4} = 3(4)^2 = 48$

$\boxed{y - 64 = 48(x - 4)}$

e) Write the equation of the normal line to the curve at $x = 4$.

$y - 64 = -\frac{1}{48}(x - 4)$

7. Let $f(x) = \sqrt{x}$

a) Find the average rate of change from $x = 4$ to $x = 9$.

$f(4) = 2$ $\qquad \dfrac{3-2}{9-4} = \dfrac{1}{5}$

$f(9) = 3$

b) Find the $\boxed{\text{instantaneous rate of change}}$ at $x = 9$.

derivative → slope at 1 pt → slope of tangent line

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \to \lim_{h\to 0} \frac{(\sqrt{x+h}-\sqrt{x})}{h} \cdot \frac{(\sqrt{x+h}+\sqrt{x})}{(\sqrt{x+h}+\sqrt{x})} \to \lim_{h\to 0} \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$$

$$\lim_{h\to 0} \frac{h}{h(\sqrt{x+h}+\sqrt{x})} \to \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{2\sqrt{x}} = f'(x)$$

$$f'(9) = \frac{1}{2\sqrt{9}} = \frac{1}{6}$$

c) Write the equation of the tangent line at $x = 9$.

$x = 9$

$y = 3$

$m = \frac{1}{6}$

$$y - 3 = \frac{1}{6}(x - 9)$$

d) Write the equation of the normal line at $x = 9$.

$$y - 3 = -6(x - 9)$$

8. An object is dropped from the top of a 150-meter tower. It's height above the ground after t seconds is given by the function $s(t) = 150 - 4.9t^2$. How fast is the object falling 2 seconds after it was dropped?

150 → initial height

$-4.9t^2$ → acceleration due to gravity part of problem

instantaneous velocity at $t = 2$

$$\lim_{h\to 0} \frac{150 - 4.9(t+h)^2 - (150 - 4.9t^2)}{h}$$

$$\frac{150 - 4.9(t^2 + 2th + h^2) - 150 + 4.9t^2}{h}$$

$$\lim_{h\to 0} \frac{150 - 4.9t^2 - 9.8th - 4.9h^2 - 150 + 4.9t^2}{h}$$

$$\lim_{h\to 0} \frac{-9.8th - 4.9h^2}{h}$$

$$\lim_{h\to 0} -9.8t - 4.9h = \boxed{-9.8t}$$

$$s'(2) = -9.8(2) = \boxed{-19.6 \text{ m/s}}$$