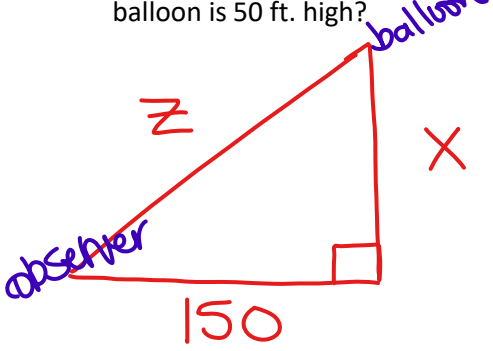
AP Calculus Related Rates Day 2 **Name:** _____

1. A small balloon is released at a point 150 feet away from an observer who is on level ground. If the balloon goes straight up at the rate of 8 ft/sec, how fast is the distance from the observer to the balloon increasing when the balloon is 50 ft. high?



$\frac{dx}{dt} = 8$ ft/sec

$x^2 + 150^2 = z^2$

$50^2 + 150^2 = z^2$

$2500 + 22500 = z^2$

$25000 = z^2$

$z = \sqrt{25,000}$

$2x\frac{dx}{dt} = 2z\frac{dz}{dt}$

$2(50)(8) = 2(\sqrt{25000})\frac{dz}{dt}$

$\frac{dz}{dt} = \frac{400}{\sqrt{25,000}}$ ft/sec

2. A man throws a stone into a still pond causing circular ripples to spread. If the radius of the circle increases at a constant rate of 1.5 ft/sec, how fast is the enclosed area of the ripples increasing when the radius of the ripple is 3 ft?

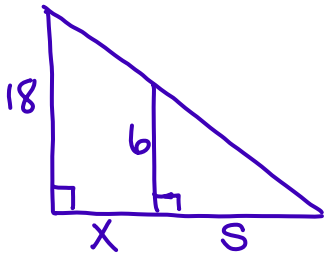
$\frac{dA}{dt}$ when $r = 3$ ft

$\frac{dr}{dt} = 15$ ft/sec

$\frac{d}{dt}[A = \pi r^2]$ $\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

$\left.\frac{dA}{dt}\right|_{r=3, dr/dt=1.5} = 2\pi(3)(1.5) = 9\pi$ ft^2/sec

3. A 6 ft. tall man is 10 ft. away from an 18 ft. light pole. The man walks towards the pole at a rate of 1 ft/sec.
   a) Find the rate at which the length of his shadow is changing.



$\frac{6}{s} = \frac{18}{x+s}$

$6x + 6s = 18s$

$6x = 12s$

$\frac{d}{dt}[x = 2s]$ $\frac{dx}{dt} = 2\frac{ds}{dt}$

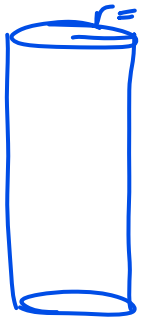
$-1 = 2\frac{ds}{dt}$

$\frac{ds}{dt} = -\frac{1}{2}$ ft/sec

   b) Find the rate at which the tip of his shadow is moving.

$-1 + \frac{-1}{2} = -1\frac{1}{2}$ ft/sec

4. A cylindrical tank of radius 10 ft. is being filled with wheat at a rate of 314 cubic feet per minute. How fast is the depth of the wheat increasing? (The volume of a cylinder is $V = \pi r^2 h$ where r is the radius and h is the height).



$\frac{dV}{dt} = 314$ ft^3/min $\frac{dh}{dt}$

$V = (\pi r^2)h$

$\frac{dV}{dt} = 2\pi r\frac{dr}{dt}\cdot h + \pi r^2\cdot\frac{dh}{dt}$

$314 = 2\pi(10)(0)\cdot h + \pi(10^2)\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{314}{100\pi}$ ft/min