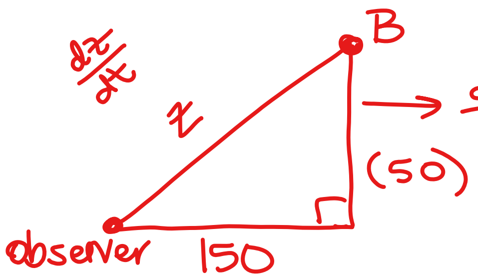
AP Calculus Related Rates Day 2 I Name: ____________________________________

1. A small balloon is released at a point 150 feet away from an observer who is on level ground. If the balloon goes straight up at the rate of 8 ft/sec, how fast is the distance from the observer to the balloon increasing when the balloon is 50 ft. high?



$\rightarrow \frac{dB}{dt} = 8$

$150^2 + B^2 = z^2 \longrightarrow 150^2 + 50^2 = z^2$

$z^2 = 25{,}000$

$z = 50\sqrt{10}$

$2B\frac{dB}{dt} = 2z\frac{dz}{dt}$

$\cancel{2}(\cancel{50})(8) = \cancel{2}(\cancel{50}\sqrt{10})\frac{dz}{dt}$

$\boxed{\frac{dz}{dt} = \frac{8}{\sqrt{10}} \text{ ft/sec}}$

2. A man throws a stone into a still pond causing circular ripples to spread. If the radius of the circle increases at a constant rate of 1.5 ft/sec, how fast is the enclosed area of the ripples increasing when the radius of the ripple is 3 ft?

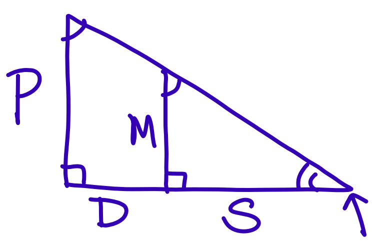
$\frac{d}{dt} A = \frac{d}{dt}\pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(3)(1.5)$

$\boxed{= 9\pi \text{ ft}^2/\text{sec}}$

3. A 6 ft. tall man is 10 ft. away from an 18 ft. light pole. The man walks towards the pole at a rate of 1 ft/sec.
   a) Find the rate at which the length of his shadow is changing.



$\frac{P}{M} = \frac{D+S}{S}$

$\frac{18}{6} = \frac{D+S}{S}$

$\rightarrow 18S = 6D + 6S$

$12S = 6D$

$2S = D$

$2\frac{dS}{dt} = \frac{dD}{dt}$

$2\frac{dS}{dt} = -1$

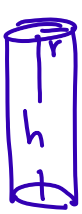
$\boxed{\frac{dS}{dt} = -\frac{1}{2} \text{ ft/s}}$

   b) Find the rate at which the tip of his shadow is moving.

Tip of shadow is moving at a rate of $-1 + -\frac{1}{2} = \boxed{-\frac{3}{2} \text{ ft/s}}$

4. A cylindrical tank of radius 10 ft. is being filled with wheat at a rate of 314 cubic feet per minute. How fast is the depth of the wheat increasing? (The volume of a cylinder is $V = \pi r^2 h$ where r is the radius and h is the height).

$V = \pi r^2 h$



$r = 10$

$\frac{dV}{dt} = 314$

$\frac{dh}{dt} = ?$

$V = \pi(10)^2 h$

$\frac{dV}{dt} = \frac{d}{dt}100\pi h$

$\frac{dV}{dt} = 100\pi\frac{dh}{dt}$

$314 = 100\pi\frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{314}{100\pi} \text{ ft/min}}$

5. If $f(x) = \ln(x+1)$, find the linearization $L(x)$ of $f(x)$ centered at $x = 0$. Use it to approximate $\ln(1.1)$ then use your calculator to determine the accuracy of the approximation.

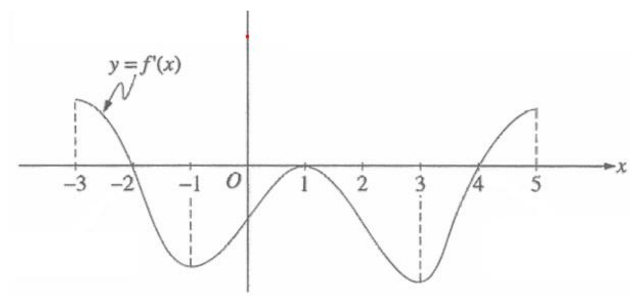
6. If $f(x) = x^2 + 2x$, find dy when $x = 0$ and $dx = 0.1$.

7. Evaluate the following limits.

a) $\lim\limits_{x\to 0^+} (\tan x)^{\sin x}$

b) $\lim\limits_{x\to 0} \dfrac{3x}{\ln(x+1)}$

8. The graph of $f'(x)$, the derivative of $f(x)$ is shown to the right. Use it to answer the following questions.



a) Find the interval for which $f(x)$ is increasing. Justify your answer.

b) Find the x-values where $f(x)$ has a relative minimum. Justify your answer.

c) Find all points of inflection for $f(x)$. Justify your answer.

d) Find the interval for which $f(x)$ is concave up. Justify your answer.