Integration Review Session Solutions

1. Evaluate

a) $\frac{d}{dx}\left[\int_0^{x^2} \ln(1+t^2)\,dt\right]$

$\ln(1+(x^2)^2)\cdot(2x) - 0$

$\boxed{2x\ln(1+x^4)}$

b) $\frac{d}{dt}\left[\int_{e^t}^{t^3} 7^x\,dx\right]$

$\boxed{7^{t^3}\cdot 3t^2 - 7^{e^t}\cdot e^t}$

2. Evaluate

a) $\int_1^2 \frac{1+y^2}{y}\,dy$

$\int_1^2 \left(\frac{1}{y}+y\right)dy$

$\ln|y| + \frac{1}{2}y^2 \Big|_1^2$

$\ln 2 + 2 - (0+\frac{1}{2})$

$\boxed{\ln 2 + \frac{3}{2}}$

b) $\int \frac{5}{\sqrt{16-49x^2}}\,dx$

$\int \frac{5}{\sqrt{16(1-\frac{49}{16}x^2)}}dx \to \frac{5}{4}\int \frac{1}{\sqrt{1-(\frac{7}{4}x)^2}}\,dx$

$u=\frac{7}{4}x \quad du=\frac{7}{4}dx \quad \frac{4}{7}du=dx$

$\frac{5}{4}\int \frac{1}{\sqrt{1-u^2}}\cdot\frac{4}{7}du \to \boxed{\frac{5}{7}\sin^{-1}\left(\frac{7}{4}x\right)+C}$

c) $\int (2+x^2)^2\,dx$

$\int (4+4x^2+x^4)\,dx$

$\boxed{4x+\frac{4}{3}x^3+\frac{1}{5}x^5+C}$

3. Evaluate

a) $\int x(x^2-4)^{\frac{7}{2}}\,dx$

$u=x^2-4 \quad du=2x\,dx \quad dx=\frac{du}{2x}$

$\int x(u)^{\frac{7}{2}}\cdot\frac{du}{2x} \to \frac{1}{2}\int u^{\frac{7}{2}}\,du$

$\frac{1}{2}\cdot\frac{2}{9}u^{\frac{9}{2}}+C$

$\boxed{\frac{1}{9}(x^2-4)^{\frac{9}{2}}+C}$

b) $\int \frac{1}{\sqrt{4-x}}\,dx$

$u=4-x \quad du=-dx$

$\int\frac{1}{\sqrt{u}}\cdot(-du)$

$-\int u^{-\frac{1}{2}}\,du \to -2u^{\frac{1}{2}}+C$

$\boxed{-2\sqrt{4-x}+C}$

c) $\int x^2e^{x^3+1}\,dx$

$u=x^3+1 \quad du=3x^2\,dx$

$dx=\frac{du}{3x^2}$

$\int x^2e^u\cdot\frac{du}{3x^2}$

$\frac{1}{3}\int e^u\,du=\frac{1}{3}e^u+C$

$\boxed{\frac{1}{3}e^{x^3+1}+C}$

4. For a certain function f, the right Riemann sum approximation of $\int_0^4 f(x)\,dx$ with n subintervals of equal length is $\frac{5n^2+4n^3+1}{12n^3+2n^2-4}$ for all n. What is the value of $\int_0^4 f(x)\,dx$?

$$\lim_{n\to\infty}\frac{5n^2+4n^3+1}{12n^3+2n^2-4}=\frac{4n^3}{12n^3}=\boxed{\frac{1}{3}}$$

Integration Review Session Solutions

5. (calculator)

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

(a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure. $\frac{55-62}{8-6} = \frac{-7}{2}$ °C/cm

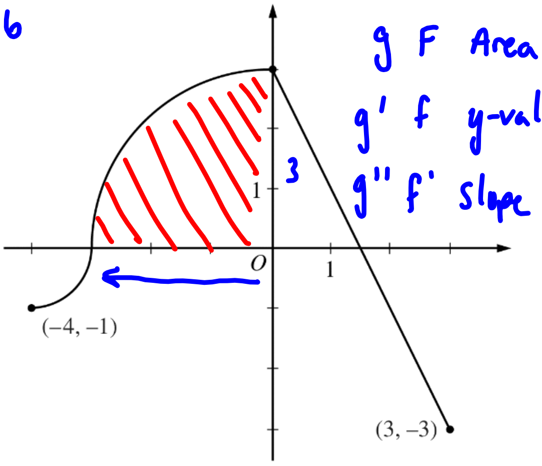
(b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure. $\frac{1}{8}\int_0^8 T(x)\,dx \approx \frac{1}{2}\cdot 1(193) + \frac{1}{2}\cdot 4(163) + \frac{1}{2}\cdot 1(132) + \frac{1}{2}\cdot 2(117) = 75.688$ °C

(c) Find $\int_0^8 T'(x)\,dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

$$\int_0^8 T'(x)\,dx = T(x)\Big|_0^8 = T(8) - T(0) = 55-100 = -45°C$$

The temperature of the other end of the wire is 45°c less than the heated end.

6



Graph of f

g	F	Area
g'	f	y-vals
g''	f'	slope

The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure.

Let $g(x) = 2x + \underline{\int_0^x f(t)\,dt}$.

$$g(-3) = 2(-3) + {}^{-}\frac{1}{4}\pi(3)^2$$

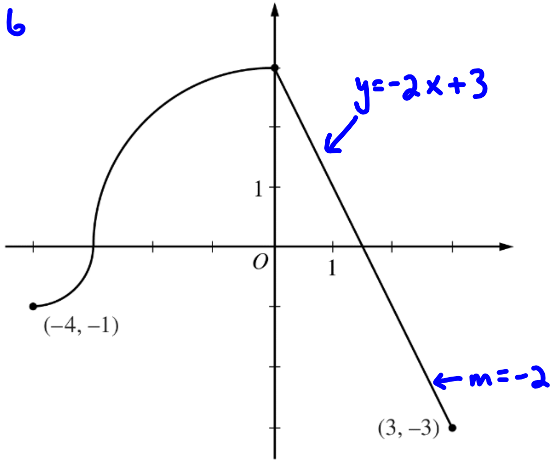
$$\boxed{g(-3) = -6 - \frac{9\pi}{4}}$$

$$\boxed{g'(x) = 2 + f(x)}$$

$$g'(-3) = 2 + f(-3) = 2+0 = \boxed{2}$$

(a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

6



Graph of f

The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure.

Let $g(x) = 2x + \int_0^x f(t)\,dt$. $g'(x) = 2 + f(x)$

crit pts: $g'(x) = 0 \rightarrow 2 + f(x) = 0$

$-2x+3=-2$ $f(x) = -2$

$-2x=-5$

$x = \frac{5}{2}$ ← critical pt

$+ \quad -$

$\frac{5}{2}$

(b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

$g(x)$ is increasing from -4 to $\frac{5}{2}$ so $g(\frac{5}{2}) > g(-4)$

$g(x)$ is decreasing from $\frac{5}{2}$ to 3 so $g(\frac{5}{2}) > g(3)$

So, Abs. max $x = \frac{5}{2}$

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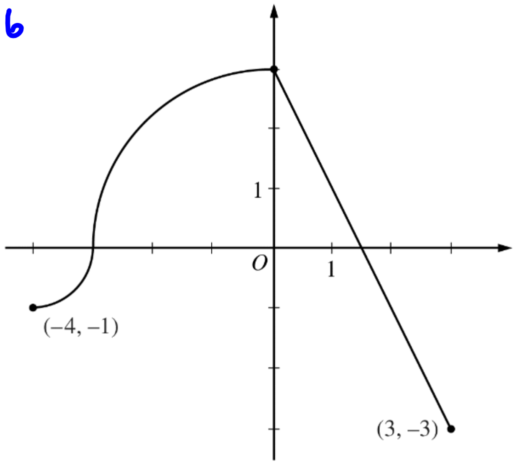

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The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure.

Let $g(x) = 2x + \int_0^x f(t)\,dt$.

$g'(x) = 2 + f(x)$

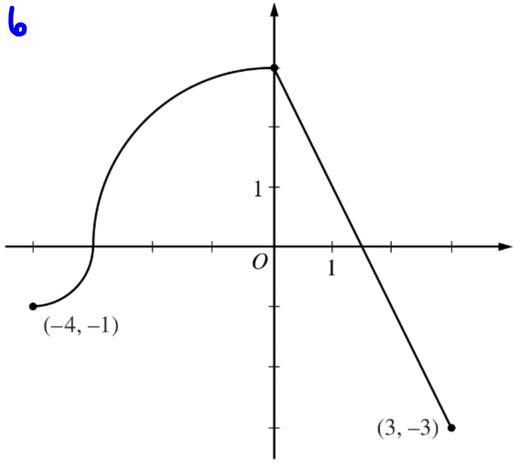
$g''(x) = f'(x)$ ← slope

(c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

Point of inflection at $x=0$ since $g''(x)$ changes sign

(slope of graph changes sign)

6



Graph of f

The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure.

Let $g(x) = 2x + \int_0^x f(t)\,dt$.

$$\frac{f(3)-f(-4)}{3--4} = \frac{-3--1}{7} = \boxed{\frac{-2}{7}}$$

f is not differentiable at x = −3 and x = 0 so MVT does not apply

(d) Find the average rate of change of f on the interval $-4 \le x \le 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

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7. The average value of a function $f(x)$ is 4 over the interval $[-1, 5]$. Find $\int_{-1}^{5} 3f(x)\,dx$.

$$6 \cdot \frac{1}{5--1}\int_{-1}^{5} f(x)\,dx = 4 \cdot 6$$

$$3\int_{-1}^{5} f(x)\,dx$$

$$\int_{-1}^{5} f(x)\,dx = 24$$

$$3 \cdot 24$$

$$\boxed{72}$$