

AB Calculus Rates of Change and Derivatives Review

Name: _____

1. Find the average rate of change of the function $f(x) = 3 + \sin x$ over the interval $[-\pi, \pi]$.
 $f(-\pi) = 3 + \sin(-\pi) = 3$
 $f(\pi) = 3 + \sin \pi = 3$
 $\frac{3-3}{\pi - (-\pi)} = 0$

2. Find the slope of the line tangent to the curve at the given value of x .
 a) $f(x) = -3x^2 + 6x$ at $x = 6$.
 $f'(x) = -6x + 6$ $f'(6) = -6(6) + 6 = -36 + 6 = -30$
 b) $f(x) = \frac{-1}{x+6}$ at $x = -4$.
 $f'(x) = \frac{1}{(x+6)^2}$ $f'(-4) = \frac{1}{(-4+6)^2} = \frac{1}{2^2} = \frac{1}{4}$
 c) $f(x) = 4 - 15x$ at $x = 3$.
 $f'(3) = -15$ $f(4) = 12$
 d) $f(x) = \begin{cases} 8+x & x \leq 4 \\ -x-6 & x > 4 \end{cases}$ at $x = 5$.
 $f'(5) = -1$ $\lim_{x \rightarrow 4^-} 8+4 = 12$

3. Find the equation of the tangent line to the curve $f(x) = \frac{7}{x} - 2$ at $(3, \frac{1}{3})$.
 $f(x) = 7x^{-1} - 2 \rightarrow f'(x) = -7x^{-2}$ $f'(3) = -\frac{7}{9}$
 $y - \frac{1}{3} = -\frac{7}{9}(x - 3)$ $-4 - 6 = -10$

4. Find the equation of the normal line to the function $y = 4x^2$ at $(3, 36)$.
 $\frac{dy}{dx} = 8x$ $\frac{dy}{dx}|_{x=3} = 8(3) = 24$ $y - 36 = -\frac{1}{24}(x - 3)$

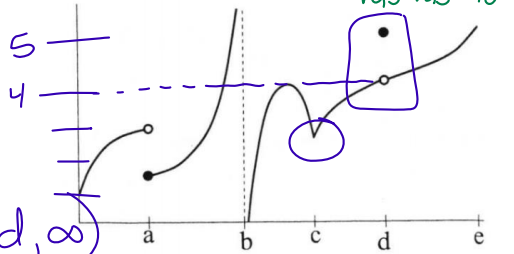
5. Find the x-value(s) where the graph of the function $f(x) = 6x^2 + 4x - 4$ has horizontal tangents.
 $f'(x) = 12x + 4$ $0 = 12x + 4$ $-4 = 12x$ $x = -\frac{1}{3}$

6. Use the limit definition of derivative to find the derivative of the function $f(x) = x^3 + 7$.
 $f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^3 + 7 - x^3 - 7}{h} ; f'(x) = 3x^2$

7. Use the alternative definition of derivative to find the derivative of each function at the indicated point.
 a) $f(x) = \sqrt{x}$ at $x = 25$.
 $\lim_{x \rightarrow 25} \frac{\sqrt{x} - 5}{x - 25} \cdot \frac{\sqrt{x} + 5}{\sqrt{x} + 5} \rightarrow \frac{x - 25}{(x - 25)(\sqrt{x} + 5)} \rightarrow \frac{1}{\sqrt{x} + 5}$
 $\frac{1}{\sqrt{25} + 5} = \frac{1}{10}$

Shortcut \rightarrow b) $f(x) = -2x^2 + 10x$ at $x = 10$.
 $f'(x) = -4x + 10$ $f'(10) = -4(10) + 10 = -30$

8. Consider the graph of f given to the right.
 a) On what interval is f continuous?
 $(-\infty, a) [a, b) (b, d) (d, \infty)$
 $\lim_{x \rightarrow d^-} f(x) = \lim_{x \rightarrow d^+} f(x) = \lim_{x \rightarrow d} f(x) = 4 \neq f(d)$

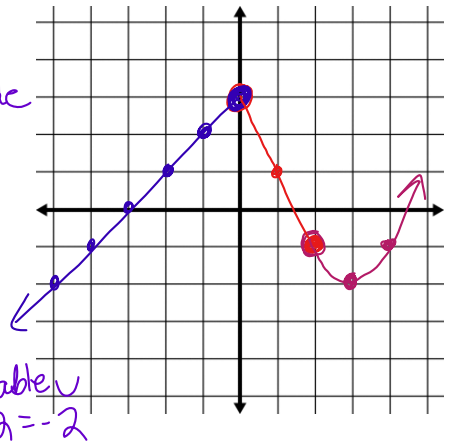


b) On what interval is f differentiable?
 $(-\infty, a) (a, b) (b, c) (c, d) (d, \infty)$

9. Graph the given function, then answer the following questions.
 $f(x) = \begin{cases} 3+x, & x \leq 0 \\ -2x+3, & 0 < x \leq 2 \\ x^2-6x+7, & x > 2 \end{cases}$
 $f'(x) = \begin{cases} 1, & x < 0 \\ -2, & 0 < x < 2 \\ 2x-6, & x > 2 \end{cases}$

a) Compare the right-hand and left-hand derivatives at $x = 0$ to prove whether or not the function is differentiable at $x = 0$. Explain your answer.
 $\lim_{x \rightarrow 0^-} f'(x) = 1$ $\lim_{x \rightarrow 0^+} f'(x) = -2$ $1 \neq -2$ Not differentiable

b) Compare the right-hand and left-hand derivatives at $x = 2$ to prove whether or not the function is differentiable at $x = 2$. Explain your answer.
 $\lim_{x \rightarrow 2^-} f'(x) = -2$ $\lim_{x \rightarrow 2^+} f'(x) = 2(2) - 6 = -2$ yes, differentiable $-2 = -2$



10. For each of the following functions, find the interval for which the function is differentiable.

a) $f(x) = \frac{1}{x^2 - 81}$

b) $f(x) = -7x + 5$

c) $f(x) = \sqrt{16 - x^2}$

11. Graph the derivative of the function below on the grid to the right.

