AB Calculus Rates of Change and Derivatives Review Name: ______________________

1. Find the average rate of change of the function $f(x) = 3 + \sin x$ over the interval $[-\pi, \pi]$.

$f(-\pi) = 3 + \sin(-\pi) = 3$ $f(\pi) = 3 + \sin(\pi) = 3$ A ROC = 0

2. Find the slope of the line tangent to the curve at the given value of x.

a) $f(x) = -3x^2 + 6x$ at $x = 6$.

$f'(x) = -6x + 6 \rightarrow f'(6) = -6(6) + 6 = -30$

b) $f(x) = \frac{-1}{x+6}$ at $x = -4$.

either Quotient Rule or $-1(x+6)^{-1} \rightarrow$ Power Rule / Chain Rule

$f'(x) = 1(x+6)^{-2}(1) \rightarrow f'(-4) = \frac{1}{4}$

chain rule (derivative of the inside $\rightarrow x+6$)

c) $f(x) = 4 - 15x$ at $x = 3$.

$f'(x) = -15$ $f'(3) = -15$

d) $f(x) = \begin{cases} 8 + x & x \le 4 \\ -x - 6 & x > 4 \end{cases}$ at $x = 5$.

$f'(5) = -1$

3. Find the equation of the tangent line to the curve $f(x) = \frac{7}{x} - 2$ at $\left(3, \frac{1}{3}\right)$.

$f(x) = 7x^{-1} - 2 \rightarrow f'(x) = -7x^{-2} \rightarrow f'(3) = -7(3)^{-2} = \frac{-7}{9}$ $y - \frac{1}{3} = \frac{-7}{9}(x-3)$

4. Find the equation of the normal line to the function $y = 4x^2$ at $(3, 36)$.

$\frac{dy}{dx} = 8x$ $\left.\frac{dy}{dx}\right|_{x=3} = 24$ $\perp \rightarrow -\frac{1}{24}$ $y - 36 = -\frac{1}{24}(x-3)$

5. Find the x-value(s) where the graph of the function $f(x) = 6x^2 + 4x - 4$ has horizontal tangents.

$f'(x) = 12x + 4$ $12x + 4 = 0$ $x = -\frac{4}{12} = \frac{-1}{3}$

6. Use the limit definition of derivative to find the derivative of the function $f(x) = x^3 + 7$.

setup $\rightarrow \lim_{h\to 0} \frac{(x+h)^3 + 7 - (x^3 + 7)}{h}$ shortcut $\rightarrow f'(x) = 3x^2$

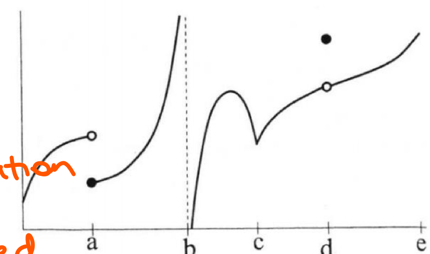
7. Use the alternative definition of derivative to find the derivative of each function at the indicated point.

a) $f(x) = \sqrt{x}$ at $x = 25$. $\lim_{x\to 25} \frac{\sqrt{x} - \sqrt{25}}{x - 25}$ $\rightarrow$ shortcut $\frac{1}{2}x^{-\frac{1}{2}} \rightarrow \frac{1}{2\sqrt{x}} \rightarrow f'(25) = \frac{1}{10}$

b) $f(x) = -2x^2 + 10x$ at $x = 10$.

$f'(x) = -4x + 10$ $f'(10) = -30$

8. Consider the graph of f given to the right.



a) On what interval is f continuous?

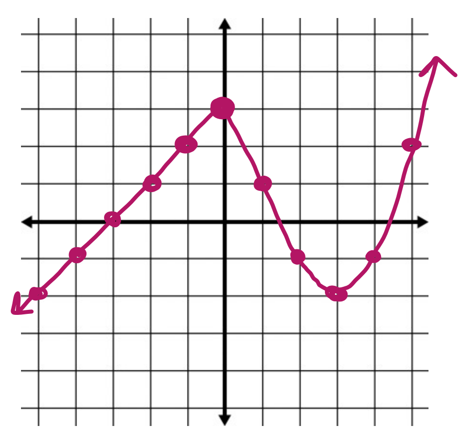
$[0, a)$ (a, b) (b, d) (d, e) or ∞ depending on interpretation

b) On what interval is f differentiable?

exact same except (b, c) (c, d) has to be separated (cusp)

9. Graph the given function, then answer the following questions.

$$f(x) = \begin{cases} 3 + x, & x \le 0 \\ -2x + 3, & 0 < x \le 2 \\ x^2 - 6x + 7, & x > 2 \end{cases}$$



① $3 + 0 = 3$
$-2(0) + 3 = 3$

$\lim_{x\to 3} f(x) = f(3)$ so continuous ✓

a) Compare the right-hand and left-hand derivatives at $x = 0$ to prove whether or not the function is differentiable at $x = 0$. Explain your answer.

$\lim_{x\to 0^-} f'(x) = 1$ $\lim_{x\to 0^+} f'(x) = -2$ Not differentiable at 0 since $1 \ne -2$

b) Compare the right-hand and left-hand derivatives at $x = 2$ to prove whether or not the function is differentiable at $x = 2$. Explain your answer. yes!

continuous at $x = 2$ $\lim_{x\to 2^-} f'(x) = -2$

$\lim_{x\to 2^+} f'(x) = 2x - 6 \rightarrow 2(2) - 6 = -2$