AB Calculus Rules for Differentiation Day 2 Practice Name: ____________________

1. What is the product rule?

$r(x) = f(x)\,g(x)$; $r'(x) = f(x)\,g'(x) + g(x)\cdot f'(x)$

2. What is the quotient rule?

$h(x) = \frac{f(x)}{g(x)}$; $h'(x) = \frac{g(x)\,f'(x) - f(x)\,g'(x)}{[g(x)]^2}$

3. Let $f(x) = (3x^3 + 4x^2)(2x^4 - 5x)$ Distribute

   a) Find $f'(x)$ without using product rule
   b) Find $f'(x)$ using product rule

$f(x) = 6x^7 - 15x^4 + 8x^6 - 20x^3$ / $f'(x) = 42x^6 - 60x^3 + 48x^5 - 60x^2$ | $f'(x) = (3x^3+4x^2)(8x^3-5) + (9x^2+8x)(2x^4-5x)$

4. Let $f(x) = \frac{x^2+4}{x} = x + \frac{4}{x} = x + 4x^{-1}$

   a) Find $f'(x)$ without using quotient rule
   b) Find $f'(x)$ using quotient rule

$f'(x) = 1 - 4x^{-2}$ or $1 - \frac{4}{x^2}$

$f'(x) = \frac{x(2x) - (x^2+4)(1)}{x^2} = \frac{2x^2 - x^2 - 4}{x^2} = \frac{x^2-4}{x^2} = 1 - \frac{4}{x^2}$

5. For each of the following, find $\frac{dy}{dx}$

   a) $y = \frac{2x-5}{3x+2}$ Quotient Rule
   b) $y = (3-x)(2+x^2)^{-1}$; $y = \frac{3-x}{2+x^2}$ Quotient Rule
   c) $y = \frac{x^3}{8-x^2}$ Quotient Rule

$\frac{dy}{dx} = \frac{(3x+2)(2) - (2x-5)(3)}{(3x+2)^2} = \frac{6x+4-6x+15}{} \rightarrow \frac{19}{(3x+2)^2}$

$\frac{dy}{dx} = \frac{(2+x^2)(-1) - (3-x)(2x)}{(2+x^2)^2} \rightarrow \frac{-2-x^2-6x+2x^2}{} \rightarrow \frac{x^2-6x-2}{(2+x^2)^2}$

6. For each of the following, write an expression for $f'(x)$ and find $f'(2)$ given the information below.

$g(2) = 3$ $\quad g'(2) = -2$

$h(2) = -1$ $\quad h'(2) = 4$

   a) $f(x) = 2g(x) + h(x)$
   b) $f(x) = 4 - h(x)$
   c) $f(x) = g(x)h(x)$
   d) $f(x) = \frac{g(x)}{h(x)}$

ⓐ $f'(x) = 2g'(x) + h'(x)$

$f'(2) = 2g'(2) + h'(2) = 2(-2) + 4 = 0$

ⓑ $f'(x) = -h'(x)$

$f'(2) = -h'(2) = -4$

ⓒ $f'(x) = g'(x)\,h(x) + h'(x)\cdot g(x)$

$f'(2) = g'(2)\,h(2) + h'(2)\,g(2) = (-2)(-1) + (4)(3) = 14$

ⓓ $f'(x) = \frac{h(x)\,g'(x) - g(x)\,h'(x)}{(h(x))^2}$

$f'(2) = \frac{(-1)(-2) - (3)(4)}{(-1)^2} = \frac{2-12}{1} = -10$

7. Suppose u and v are differentiable functions at $x = 3$ and $u(3) = 4$, $\left.\frac{du}{dx}\right|_{x=3} = -3$, $v(3) = 2$ and $\left.\frac{dv}{dx}\right|_{x=3} = 3$. Find the following at $x = 3$.

   a) $\frac{d}{dx}\left[\frac{u}{v}\right]$
   b) $\frac{d}{dx}[uv]$
   c) $\frac{d}{dx}[5u - 2v + 4\underbrace{uv}_{\text{product Rule}}]$
   d) $\frac{d}{dx}\left[\frac{v}{u}\right]$

ⓒ $5\frac{du}{dx} - 2\frac{dv}{dx} + 4\left(u\frac{dv}{dx} + v\frac{du}{dx}\right)$

$5(-3) - 2(3) + 4(4\ 3 + 2\ -3) = -15 - 6 + 4(12-6) = -21 + 24 = \boxed{3}$

8. Find the values of a and b so that $g(x)$ is both continuous and differentiable at $x = 1$.

$$g(x) = \begin{cases} x^2 + 2, & x \le 1 \\ a\left(x - \frac{1}{x}\right) + b, & x > 1 \end{cases}$$

$b \rightarrow 3$

$a(x - 1x^{-1}) + 3$

$g'(x) = \begin{cases} 2x \\ a(1 + 1x^{-2}) \end{cases}$

$1^2 + 2 = a\left(1 - \frac{1}{1}\right) + b$

$3 = a(0) + b$

$\boxed{3 = b}$

$2(1) = a(1 + 1(1)^{-2})$

$2 = 2a$

$\boxed{a = 1}$

9. At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent line parallel to the line $2x - 4y = 3$?

derivative ; same slope ; put in slope intercept form

$\frac{dy}{dx} = 2\cdot\frac{1}{2}x^{2-1} = x \rightarrow x = \frac{1}{2}$

$\frac{-4y}{-4} = \frac{-2x}{-4} + \frac{3}{-4}$

$m = \frac{1}{2}$

$\rightarrow y\left(\frac{1}{2}\right) = \frac{1}{2}\left(\frac{1}{2}\right)^2 = \frac{1}{8}$ $\quad \left(\frac{1}{2}, \frac{1}{8}\right)$