AB Calculus The Derivative of a Function Part 2 Homework Name: ____________________

1. How do you find average rate of change?

$\frac{f(b)-f(a)}{b-a}$ / slope of a secant line / rate of change between 2 pts

2. How do you find instantaneous rate of change?

$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ / Slope of a tangent line / rate at 1 pt $\Rightarrow$ derivative / $\lim_{x\to c} \frac{f(x)-f(c)}{x-c}$

3. Use the original limit definition of derivative to find the derivative of each of the following.

a) $f(x) = x^3 + x$

$f(x+h) = (x+h)^3 + (x+h)$

$\frac{f(x+h)-f(x)}{h} = \frac{\cancel{x^3}+3x^2\cancel{h}+3xh^{\cancel{2}}+h^{\cancel{3}\,2}+\cancel{x}+\cancel{h}^{+1}-\cancel{x^3}-\cancel{x}}{\cancel{h}}$

$\lim_{h\to 0} 3x^2+3xh+h^2+1 = \boxed{3x^2+1=f'(x)}$

b) $f(x) = \sqrt{x+2}$

$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \to \lim_{h\to 0} \frac{\sqrt{x+h+2}-\sqrt{x+2}}{h} \cdot \frac{\sqrt{x+h+2}+\sqrt{x+2}}{\sqrt{x+h+2}+\sqrt{x+2}}$

$\lim_{h\to 0} \frac{x+h+2-x-2}{h(\sqrt{x+h+2}+\sqrt{x+2})}$ $\lim_{h\to 0} \frac{\cancel{h}\ 1}{\cancel{h}(\sqrt{x+h+2}+\sqrt{x+2})} = \boxed{\frac{1}{2\sqrt{x+2}} = f'(x)}$

4. Use the alternative definition of the derivative to find the derivative of each of the following.

regular limit def

a) $f(x) = \frac{1}{2x-3}$ $f(x+h) = \frac{1}{2(x+h)-3} = \frac{1}{2x+2h-3}$

$\lim_{h\to 0} \frac{\frac{(2x-3)}{(2x-3)}\frac{1}{(2x+2h-3)} - \frac{1}{(2x-3)}\cdot\frac{(2x+2h-3)}{(2x+2h-3)}}{h}$

$\lim_{h\to 0} \frac{\cancel{2x}-\cancel{3}-\cancel{2x}-2\cancel{h}+\cancel{3}}{\cancel{h}(2x-3)(2x+2h-3)} \Rightarrow \lim_{h\to 0} \frac{-2}{(2x-3)(2x+2h-3)} = \boxed{\frac{-2}{(2x-3)^2} = f'(x)}$

b) $f(x) = 4 + 2x + x^2$ at $x = 3$.

$\lim_{x\to c} \frac{f(x)-f(c)}{x-c}$ $\lim_{x\to 3} \frac{f(x)-f(3)}{x-3}$ $\lim_{x\to 3} \frac{4+2x+x^2-19}{x-3}$

$f(3)=19$ $\lim_{x\to 3} \frac{f(x)-19}{x-3}$ $\lim_{x\to 3} \frac{x^2+2x-15}{x-3}$

$\lim_{x\to 3} \frac{(x+5)\cancel{(x-3)}}{\cancel{x-3}} \to 8$ $f'(3)=8$

5. Find the derivative of the function $2x^2 - 13x + 5$ and use it to find the equation of the tangent line to the curve $x = 3$.

$$\lim_{h\to 0} \frac{2(x+h)^2-13(x+h)+5-(2x^2-13x+5)}{h}$$

$$\frac{2(x^2+2xh+h^2)-13x-13h+5-2x^2+13x-5}{h}$$

$$\frac{\cancel{2x^2}+4x\cancel{h}+2h^{\cancel{2}}-\cancel{13x}-13\cancel{h}+\cancel{5}-\cancel{2x^2}+\cancel{13x}-\cancel{5}}{\cancel{h}}$$

$$\lim_{h\to 0} 4x+2h-13$$

$f'(x)=4x-13$

$f'(3)=4(3)-13=-1$

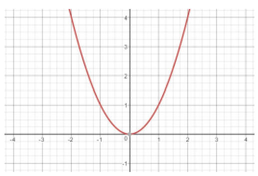
$f(3)=2(3)^2-13(3)+5=-16$

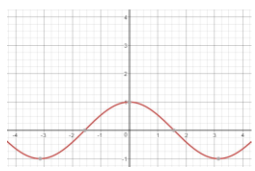
$$\boxed{y+16=-1(x-3)}$$

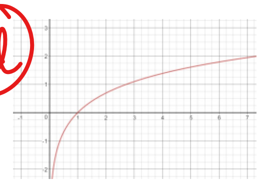
6. If $f'(4) = -3$ and $f(4) = 7$, find the equation of the normal line to $f(x)$ at $x = 4$.

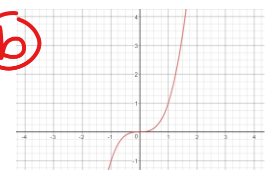
$$y-7=\frac{1}{3}(x-4)$$

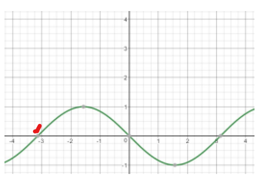
7. Match the graph of each function in the top row with the graph of its derivative in the bottom row.

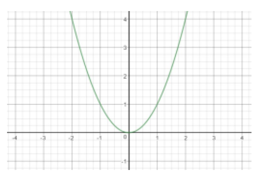
I. c


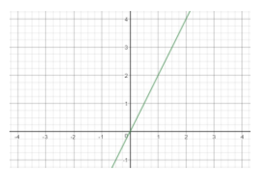
II. a


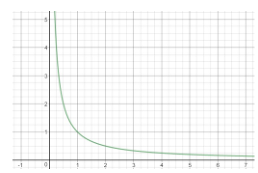
III. d


IV. b


a)


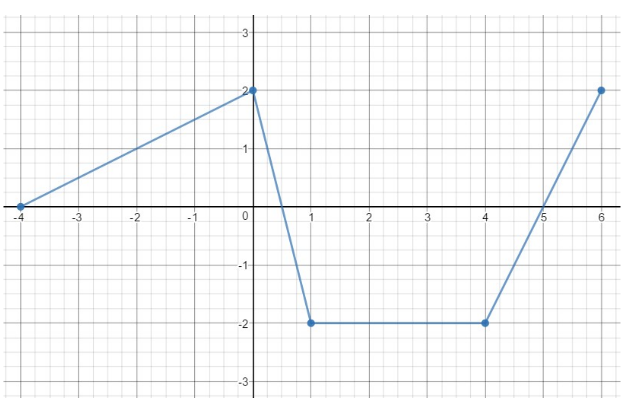
b)


c)


d)


8. The graph of the function $y = f(x)$ shown here is made of line segments joined end to end. Graph $f'(x)$ in the space provided below.

Graph of $f(x)$



Graph $f'(x)$ here