

1. How do you find average rate of change?

$$\frac{y_2 - y_1}{x_2 - x_1} \text{ or } \frac{f(b) - f(a)}{b - a}$$

2. How do you find instantaneous rate of change?

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = f'(x) = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$$

3. Use the original limit definition of derivative to find the derivative of each of the following.

a)  $f(x) = x^3 + x$

$$\lim_{h \rightarrow 0} \frac{(x+h)^3 + x+h - x^3 - x}{h}$$

$$\frac{x^3 + 3x^2h + 3xh^2 + h^3 + x+h - x^3 - x}{h}$$

$$\lim_{h \rightarrow 0} \frac{3x^2 + 3xh + h^2 + 1}{h}$$

$$f'(x) = 3x^2 + 1$$

b)  $f(x) = \sqrt{x+2}$

$$\lim_{h \rightarrow 0} \frac{\sqrt{x+h+2} - \sqrt{x+2}}{h} \cdot \frac{\sqrt{x+h+2} + \sqrt{x+2}}{\sqrt{x+h+2} + \sqrt{x+2}}$$

$$\frac{x+h+2 - x-2}{h(\sqrt{x+h+2} + \sqrt{x+2})} \rightarrow \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h+2} + \sqrt{x+2}} \Rightarrow f'(x) = \frac{1}{2\sqrt{x+2}}$$

4. Use the alternative definition of the derivative to find the derivative of each of the following.

a)  $f(x) = \frac{1}{2x-3}$  at  $x=2$

Ans  $f'(2) = -2$

$$f(2) = \frac{1}{4-3} = 1$$

$$\lim_{x \rightarrow 2} \frac{\frac{1}{2x-3} - 1}{x-2} \cdot \frac{(2x-3)}{(2x-3)} \rightarrow \frac{1-2x+3}{(x-2)(2x-3)} \rightarrow \frac{-2x+4}{(x-2)(2x-3)} \cdot \frac{-2(x-2)}{-2(x-2)}$$

b)  $f(x) = 4 + 2x + x^2$  at  $x=3$ .  $f(3) = 4 + 6 + 9$

$$\lim_{x \rightarrow 3} \frac{4 + 2x + x^2 - 19}{x-3}$$

$$\lim_{x \rightarrow 3} \frac{x^2 + 2x - 15}{x-3}$$

$$\lim_{x \rightarrow 3} \frac{(x+5)(x-3)}{x-3}$$

$$\lim_{x \rightarrow 3} x+5 = 8$$

Ans.  $f'(3) = 8$

5. Find the derivative of the function  $2x^2 - 13x + 5$  and use it to find the equation of the tangent line to the curve  $x = 3$ .

$$f'(x) = 4x - 13$$

$$f'(3) = 4(3) - 13 = -1$$

$$y + 16 = -1(x - 3)$$

$$f(3) = 2(3)^2 - 13(3) + 5 = -16$$

6. If  $f'(4) = -3$  and  $f(4) = 7$ , find the equation of the normal line to  $f(x)$  at  $x = 4$ .

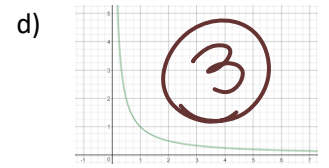
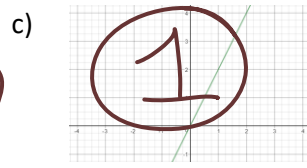
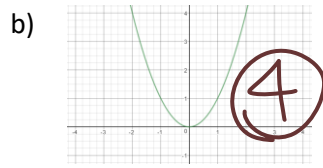
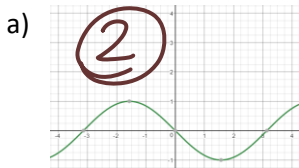
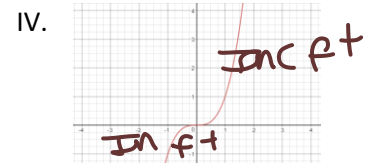
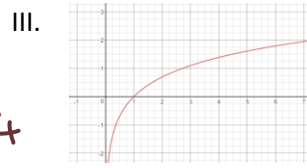
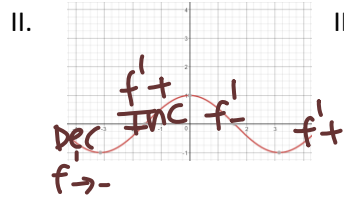
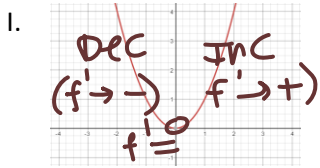
Tangent

$$y - 7 = -3(x - 4)$$

Normal

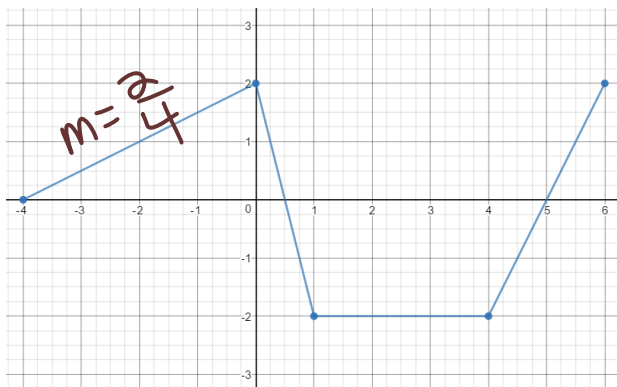
$$y - 7 = \frac{1}{3}(x - 4)$$

7. Match the graph of each function in the top row with the graph of its derivative in the bottom row.



8. The graph of the function  $y = f(x)$  shown here is made of line segments joined end to end. Graph  $f'(x)$  in the space provided below.

Graph of  $f(x)$



Graph  $f'(x)$  here

