2. Evaluate each of the following definite integrals.

$u = x^2 + 1$

$\frac{du}{dx} = 2x$

$\frac{1}{2} du = x\,dx$

$u = (1)^2 + 1 = 2$

$u = (-1)^2 + 1 = 2$

a) $\int_{-1}^{1} x(x^2+1)^3 dx$

$\frac{1}{2}\int_2^2 u^3 \, du = \boxed{0}$

b) $\int_1^2 2x^2\sqrt{x^3+1}dx$

$u = x^3 + 1$

$\frac{du}{dx} = 3x^2$

$\frac{1}{3} du = x^2 dx$

$u = 2^3 + 1 = 9$

$u = 1^3 + 1 = 2$

$\frac{2}{3}\int_2^9 u^{\frac{1}{2}} \, du$

$\frac{2}{3} \frac{2}{3} u^{\frac{3}{2}} \Big|_2^9 = \frac{4}{9}(9)^{\frac{3}{2}} - \frac{4}{9}(2)^{\frac{3}{2}}$

$\frac{4}{9}(27) - \frac{4}{9} 2\sqrt{2} = \boxed{12 - \frac{8\sqrt{2}}{9}}$

c) $\int_0^4 \frac{1}{\sqrt{2x+1}} dx$ 2

d) $\int_1^9 \frac{1}{\sqrt{x}(1+\sqrt{x})^2} dx$

$u = 1 + \sqrt{x}$

$u = 1 + \sqrt{9} = 4$

$u = 1 + \sqrt{1} = 2$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$2\,du = \frac{1}{\sqrt{x}} dx$

$2\int_2^4 \frac{1}{u^2} \, du$

$2\int_2^4 u^{-2} du = \frac{2}{-1} u^{-1} \Big|_2^4 = -2(4)^{-1} + 2(2)^{-1}$

$= -\frac{2}{4} + \frac{2}{2} = \frac{1}{2}$

$u = 1 - x$

$\frac{du}{dx} = -1$

$\frac{1}{-1} du = dx$

e) $\int_1^2 e^{1-x} dx$

$u = 1 - 2 = -1$

$u = 1 - 1 = 0$

$-1\int_0^{-1} e^u \, du$

$= -1 e^u \Big|_0^{-1} = -1 e^{-1} + e^0 = -\frac{1}{e} + 1$

f) $\int_1^e \frac{(1+\ln x)^2}{x} dx$

$u = 1 + \ln x$

$\frac{du}{dx} = \frac{1}{x} \rightarrow du = \frac{1}{x} dx$

$u = 1 + \ln e = 1 + 1 = 2$

$u = 1 + \ln 1 = 1 + 0 = 1$

$\int_1^2 u^2 \, du = \frac{1}{3} u^3 \Big|_1^2 = \frac{1}{3}(2)^3 - \frac{1}{3}(1)^3$

$= \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$