AB Calculus U-Substitution Day 2 **Name:** ______________________________

1. Evaluate each indefinite integral.

a) $\int 5e^{5x}\,dx$

$u = 5x$

$\frac{du}{dx} = 5$

$\frac{du}{5} = dx$

$5\int e^{5x}\,dx$

$\frac{\cancel{5}}{\cancel{5}}\int e^u\,du$

$e^u + C \rightarrow \boxed{e^{5x} + C}$

b) $\int \sqrt{2x-1}\,dx$

$u = 2x - 1$

$\frac{du}{dx} = 2$

$\frac{du}{2} = dx$

$\int (2x-1)^{\frac{1}{2}}\,dx$

$\frac{1}{2}\cdot\frac{2}{3}(2x-1)^{\frac{3}{2}} + C$

c) $\int \sin^3 x\,\cancel{\cos x\,dx}$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x\,dx$

$\int u^3\,du$

$\frac{1}{4}u^4 + C \rightarrow \boxed{\frac{1}{4}\sin^4 x + C}$

d) $\int \cancel{t}\sqrt{t^2+2}\,\cancel{dt}$

$u = t^2 + 2$

$\frac{du}{dt} = 2t$

$\frac{du}{2} = t\,dt$

$\frac{1}{2}\int u^{\frac{1}{2}}\,du$

$\frac{1}{2}\cdot\frac{2}{3}u^{\frac{3}{2}} + C$

$\boxed{\rightarrow \frac{1}{3}(t^2+2)^{\frac{3}{2}} + C}$

e) $\int \tan^2 x \sec^2 x\,dx$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$du = \sec^2 x\,dx$

$\int u^2\,du$

$\frac{1}{3}u^3 + C \rightarrow \boxed{\frac{1}{3}\tan^3 x + C}$

f) $\int x^2(6-x^3)^4\,dx$

$u = 6 - x^3$

$\frac{du}{dx} = -3x^2$

$\frac{du}{-3} = x^2\,dx$

$-\frac{1}{3}\int u^4\,du$

$-\frac{1}{3}\frac{u^5}{5} + C$

$\boxed{\rightarrow -\frac{1}{15}(6-x^3)^5 + C}$

g) $\int \frac{x^2}{(1+x^3)^2}\,dx$

$u = 1 + x^3$

$\frac{du}{dx} = 3x^2$

$\frac{du}{3} = x^2\,dx$

$\frac{1}{3}\int \frac{1}{u^2}\,du$

$\frac{1}{3}\int u^{-2}\,du$

$\rightarrow \frac{1}{3}\frac{u^{-1}}{-1} + C \rightarrow \boxed{-\frac{1}{3}(1+x^3)^{-1} + C}$

h) $\int \frac{x^3}{\sqrt{1+x^4}}\,dx$

$u = 1 + x^4$

$\frac{du}{dx} = 4x^3$

$\frac{du}{4} = x^3\,dx$

$\frac{1}{4}\int \frac{1}{\sqrt{u}}\,du$

$\rightarrow \frac{1}{4}\int u^{-\frac{1}{2}}\,du$

$\frac{1}{4}\,2u^{\frac{1}{2}} + C = \boxed{\frac{1}{2}(1+x^4)^{\frac{1}{2}} + C}$

2. Evaluate each of the following definite integrals.

$u=1^3+1=2$
$u=(-1)^2+1=2$

a) $\int_{-1}^{1} x(x^2+1)^3\,dx$

$u=x^2+1$

$\frac{du}{dx}=2x$

$\frac{du}{2}=x\,dx$

$\frac{1}{2}\int_2^2 u^3\,du$

$\boxed{=0}$

$u=2^3+1$ $\quad u=1^3+1$

b) $\int_1^2 2x^2\sqrt{x^3+1}\,dx$

$u=x^3+1$

$\frac{du}{dx}=3x^2$

$\frac{du}{3}=x^2dx$

$\frac{2}{3}\int_2^9 u^{\frac{1}{2}}\,du$

$\frac{2}{3}\,\frac{2}{3}u^{\frac{3}{2}}\Big|_2^9=\frac{4}{9}(9)^{\frac{3}{2}}-\frac{4}{9}(2)^{\frac{3}{2}}$

$u=2(4)+1=9$
$u=2(0)+1=1$

c) $\int_0^4 \frac{1}{\sqrt{2x+1}}\,dx$

$u=2x+1$

$\frac{du}{dx}=2$

$\frac{du}{2}=dx$

$\frac{1}{2}\int_1^9 u^{-\frac{1}{2}}\,du$

$2\,\frac{1}{2}u^{\frac{1}{2}}\Big|_1^9=\sqrt{9}-\sqrt{1}=2$

since this is $u^{-\frac{1}{2}}$
$\int u^{-\frac{1}{2}}\to 2u^{\frac{1}{2}}$

$u=1+\sqrt{9}=4$
$u=1+\sqrt{1}$

d) $\int_1^9 \frac{1}{\cancel{\sqrt{x}}(1+\sqrt{x})^2}\cancel{dx}$

$u=1+\sqrt{x}$

$\frac{du}{dx}=\frac{1}{2\sqrt{x}}$

$2du=\frac{1}{\sqrt{x}}dx$

$2\int_2^4 \frac{1}{u^2}\,du$

$2\int_2^4 u^{-2}du\to\frac{2u^{-1}}{-1}\Big|_2^4$

$=-2(4)^{-1}+2(2)^{-1}=-\frac{2}{4}+\frac{2}{2}=\frac{1}{2}$

$u=1-2=-1$
$u=1-1=0$

e) $\int_1^2 e^{1-x}\,dx$

$u=1-x$

$\frac{du}{dx}=-1$

$\frac{du}{-1}=dx$

$-1\int_0^{-1} e^u\,du$

$-1e^u\Big|_0^{-1}=-1e^{-1}+1e^0=\boxed{-\frac{1}{e}+1}$

f) $\int_1^e \frac{(1+\ln x)^2}{x}\,dx$

$u=1+\ln x$

$\frac{du}{dx}=\frac{1}{x}$

$du=\frac{1}{x}dx$

$u=1+\ln e=2$
$u=1+\ln 1=1$

$\int_1^2 u^2\,du$

$\frac{u^3}{3}\Big|_1^2=\frac{2^3}{3}-\frac{1^3}{3}$

$=\frac{8}{3}-\frac{1}{3}$

$=\frac{7}{3}$