

Free Response Practice 1 (no calculator)

Consider the curve given by $xy^2 - x^3y = 6$.

- Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.
- Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.
- Find the x-coordinate of each point on the curve where the tangent line is vertical.

(a) $\frac{d}{dx} [xy^2 - x^3y = 6]$

$$x \cdot 2yy' + y^2 + \underline{-x^3y'} + y(-3x^2) = 0$$

$$2xyy' - x^3y' = 3x^2y - y^2$$

$$y'(2xy - x^3) = 3x^2y - y^2$$

$$y' = \frac{3x^2y - y^2}{2xy - x^3} \rightarrow \boxed{\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}}$$

(b) $(1)(y^2) - 1(y) = 6$

$$y^2 - y - 6 = 0$$

$$(y-3)(y+2) = 0$$

$$y=3 \quad y=-2$$

$$(1, 3)$$

$$(1, -2)$$

$$\frac{dy}{dx} = \frac{3(3) - 3^2}{2(3) - 1}$$

$$\frac{dy}{dx} = \frac{3(-2) - (-2)^2}{2(-2) - 1} = \frac{-10}{-5} = 2$$

$$\frac{dy}{dx} = 0$$

$$y - 3 = 0(x - 1)$$

OR
 $\boxed{y=3}$

$$\boxed{y+2 = 2(x-1)}$$

(c) $2xy - x^3 = 0$

$$x(2y - x^2) = 0$$

$$x \neq 0$$

$$2y - x^2 = 0$$

$$2y = x^2$$

$$y = \frac{x^2}{2}$$

$$xy^2 - x^3y = 6$$

$$x\left(\frac{x^2}{2}\right)^2 - x^3\left(\frac{x^2}{2}\right) = 6$$

$$x\left(\frac{x^4}{4}\right) - \frac{x^5}{2} = 6$$

$$4 \cdot \left[\frac{x^5}{4} - \frac{x^5}{2} = 6 \right] \cdot 4$$

$$x^5 - 2x^5 = 24$$

$$-x^5 = 24$$

$$x^5 = -24$$

$$x = \sqrt[5]{-24}$$

Free Response Practice 2 (no calculator)

Let f be a function given by $f(x) = \ln\left(\frac{x}{x-1}\right)$.

- (a) What is the domain of f ?
- (b) Write an expression for the general derivative of $f(x)$.
- (c) Write an equation for the line tangent to $f(x)$ at $x = -1$.

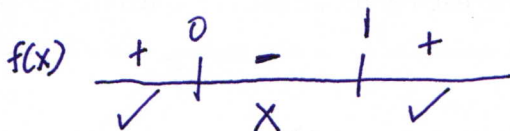
(a)

$$\frac{x}{x-1} > 0$$

$$x \neq 1, x \neq 0$$

$$(0, 1) \cup (1, \infty)$$

$$\boxed{(-\infty, 0) \cup (1, \infty)}$$



(b)

$$f(x) = \ln\left(\frac{x}{x-1}\right)$$

$$f(x) = \ln x - \ln(x-1)$$

$$f'(x) = \frac{1}{x} - \frac{1}{x-1}$$

(c)

$$f'(-1) = -1 - \frac{1}{-2}$$

$$f'(-1) = -\frac{1}{2}$$

$$f(-1) = \ln\left(\frac{-1}{-1-1}\right)$$

$$f(-1) = \ln\left(\frac{1}{2}\right) = \ln 1 - \ln 2 = 0 - \ln 2 = -\ln 2$$

$$\boxed{y - \ln\left(\frac{1}{2}\right) = -\frac{1}{2}(x + 1)}$$

$$y + \ln 2 = -\frac{1}{2}(x + 1)$$