**Free Response Practice 1 (no calculator)**

Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.

(b) Find all points on the curve whose $x$-coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the $x$-coordinate of each point on the curve where the tangent line is vertical.

(a) $\frac{d}{dx}[xy^2 - x^3y = 6]$

$x \cdot 2yy' + y^2 + -x^3y' + y(-3x^2) = 0$

$2xyy' - x^3y' = 3x^2y - y^2$

$y'(2xy - x^3) = 3x^2y - y^2$

$y' = \frac{3x^2y - y^2}{2xy - x^3} \Rightarrow \boxed{\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}}$

(b) $(1)(y^2) - 1(y) = 6$

$y^2 - y - 6 = 0$

$(y-3)(y+2) = 0$

$y = 3 \quad y = -2$

$(1, 3)$

$\frac{dy}{dx} = \frac{3(3) - 3^2}{2(3) - 1}$

$\frac{dy}{dx} = 0$

$y - 3 = 0(x-1)$

OR

$\boxed{y = 3}$

$(1, -2)$

$\frac{dy}{dx} = \frac{3(-2) - (-2)^2}{2(-2) - 1} = \frac{-10}{-5} = 2$

$\boxed{y + 2 = 2(x-1)}$

(c) $2xy - x^3 = 0$

$x(2y - x^2) = 0$

$x \neq 0 \quad 2y - x^2 = 0$

$2y = x^2$

$y = \frac{x^2}{2}$

$xy^2 - x^3y = 6$

$x\left(\frac{x^2}{2}\right)^2 - x^3\left(\frac{x^2}{2}\right) = 6$

$x\left(\frac{x^4}{4}\right) - \frac{x^5}{2} = 6$

$4 \cdot \left[\frac{x^5}{4} - \frac{x^5}{2} = 6\right] \cdot 4$

$x^5 - 2x^5 = 24$

$-x^5 = 24$

$x^5 = -24$

$\boxed{x = \sqrt[5]{-24}}$

**Free Response Practice 2 (no calculator)**

Let $f$ be a function given by $f(x)=\ln\left(\frac{x}{x-1}\right)$.

(a) What is the domain of $f$?
(b) Write an expression for the general derivative of $f(x)$.
(c) Write an equation for the line tangent to $f(x)$ at $x=-1$.

(a) $\frac{x}{x-1} > 0$

$f(x)$: + 0 − | 1 +

$x \neq 1$, $x \neq 0$

~~$(0,1)$~~ $(1,\infty)$

$(-\infty,0) \cup (1,\infty)$

(b) $f(x) = \ln\left(\frac{x}{x-1}\right)$

$f(x) = \ln x - \ln(x-1)$

$f'(x) = \frac{1}{x} - \frac{1}{x-1}$

(c) $f'(-1) = -1 - \frac{1}{-2}$

$f'(-1) = -\frac{1}{2}$

$f(-1) = \ln\left(\frac{-1}{-1-1}\right)$

$f(-1) = \ln\left(\frac{1}{2}\right) = \ln 1 - \ln 2 = 0 - \ln 2 = -\ln 2$

$y - \ln\left(\frac{1}{2}\right) = -\frac{1}{2}(x+1)$

$y + \ln 2 = -\frac{1}{2}(x+1)$