Pre-Calculus Name________________________________



# Addition and Subtraction Formulas Notes

Date________________ Period____

**Use the angle sum identity to find the exact value of each.**

1) $\sin 195°$

$\sin(150° + 45°)$

$\sin 150° \cos 45° + \sin 45° \cos 150°$

$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

2) $\cos 75°$ $\cos(30° + 45°)$

$\cos 30° \cos 45° - \sin 30° \sin 45°$

$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

**Use the angle difference identity to find the exact value of each.**

3) $\tan \frac{17\pi}{12} = \frac{\boxed{7}\pi}{4}\cdot\frac{3}{3} - \frac{\boxed{1}\pi}{3}\cdot\frac{4}{4}$

$\tan\left(\frac{7\pi}{4} - \frac{\pi}{3}\right)$

$= \frac{\tan\frac{7\pi}{4} - \tan\frac{\pi}{3}}{1 + \tan\left(\frac{7\pi}{4}\right)\tan\left(\frac{\pi}{3}\right)} = \frac{(-1-\sqrt{3})}{(1-\sqrt{3})} \cdot \frac{(1+\sqrt{3})}{(1+\sqrt{3})}$

$= \frac{-1-\sqrt{3}-\sqrt{3}-3}{1-3} = \frac{-4-2\sqrt{3}}{-2} = \boxed{2+\sqrt{3}}$

4) $\cos -\frac{13\pi}{12} = \frac{\boxed{1}\pi}{4}\cdot\frac{3}{3} - \frac{\boxed{4}\pi}{3}\cdot\frac{4}{4}$

$\cos\left(\frac{\pi}{4} - \frac{4\pi}{3}\right)$

$= \cos\frac{\pi}{4}\cos\frac{4\pi}{3} + \sin\frac{\pi}{4}\sin\frac{4\pi}{3}$

$= \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right)$

$= -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{-\sqrt{2}-\sqrt{6}}{4}}$

**Use an addition or subtraction formula to write the expression as a trigonometric function of one number, and then find its exact value.**

5) $\cos 10° \cos 80° - \sin 10° \sin 80°$

$\cos(10° + 80°)$

$\cos 90°$

$\boxed{= 0}$

6) $\sin 23° \cos 37° + \sin 37° \cos 23°$

$\sin(23° + 37°)$

$\sin 60°$

$\boxed{= \frac{\sqrt{3}}{2}}$

7) $\cos \frac{19\pi}{12} \cos \frac{\pi}{4} + \sin \frac{19\pi}{12} \sin \frac{\pi}{4}$

$\cos\left(\frac{19\pi}{12} - \frac{\pi}{4} \cdot \frac{3}{3}\right)$

$\cos\left(\frac{19\pi - 3\pi}{12}\right)$

$\cos\left(\frac{16\pi}{12}\right) = \cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$

8) $(\tan \pi + \tan \frac{\pi}{6}) \div (1 - \tan \pi \tan \frac{\pi}{6})$

$\tan\left(\pi + \frac{\pi}{6}\right)$

$\tan\left(\frac{7\pi}{6}\right)$

$= \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

**Prove the identity.**

9) $\cos(\theta - \frac{\pi}{2}) = \sin \theta$

$\cos\theta \cos\frac{\pi}{2} + \sin\theta \cdot \sin\frac{\pi}{2}$

$\cos\theta \cdot 0 + \sin\theta \quad 1$

$\boxed{= \sin\theta}$

10) $\sin(\frac{\pi}{2} - \theta) = \cos \theta$

$\sin\frac{\pi}{2} \cos\theta - \sin\theta \cos\frac{\pi}{2}$

$1 \cos\theta - \sin\theta \quad 0$

$\boxed{= \cos\theta}$

11) $\cot(x+y) = (\cot x \cot y - 1) \div (\cot x + \cot y)$

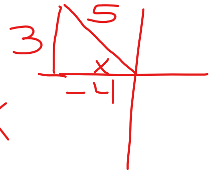
$$\frac{1}{\tan(x+y)}$$

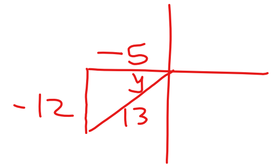
$$\frac{1-\tan x \tan y}{\tan x+\tan y}$$

$$\frac{\frac{\cot x \cdot \cot y}{\cot x \cot y} - \frac{1}{\cot x}\frac{1}{\cot y}}{\frac{\cot y}{\cot y}\frac{1}{\cot x} + \frac{1}{\cot y}\frac{\cot x}{\cot x}} = \frac{\cot x \cot y - 1}{\cot y + \cot x} = \text{RHS} \checkmark$$

**Assuming** $\sin x = \frac{3}{5}$ **if** $\frac{\pi}{2} < x < \pi$ **and** $\cos y = -\frac{5}{13}$ **if** $\pi < y < \frac{3\pi}{2}$




12) $\sin(x + y)$

$\sin x \cos y + \sin y \cos x$

$\left(\frac{3}{5}\right)\left(\frac{-5}{13}\right) + \left(\frac{-12}{13}\right)\left(\frac{-4}{5}\right)$

$= \frac{-15}{65} + \frac{48}{65}$

$\boxed{= \frac{33}{65}}$

13) $\cos(x + y)$

$\cos x \cos y - \sin x \sin y$

$\left(\frac{-4}{5}\right)\left(\frac{-5}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{-12}{13}\right)$

$= \frac{20}{65} + \frac{36}{65}$

$\boxed{= \frac{56}{65}}$

14) $\tan(x - y)$

$\frac{\tan x - \tan y}{1 + \tan x \tan y}$

$= \frac{\frac{5}{5}\,\frac{-3}{4} - \frac{12}{5}\,\frac{4}{4}}{\frac{20}{20}\; 1 + \left(\frac{-3}{4}\right)\left(\frac{12}{5}\right)}$

$= \frac{-15 - 48}{20 - 36} = \boxed{\frac{+63}{+16}}$

15) $\sec(x - y)$

$\frac{1}{\cos(x-y)} = \frac{1}{\cos x \cos y + \sin x \sin y}$

$= \frac{1}{\left(-\frac{4}{5}\right)\left(-\frac{5}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{-12}{13}\right)}$

$= \frac{1}{\frac{20}{65} - \frac{36}{65}}$

$= \frac{1}{\frac{-16}{65}} = -\frac{65}{16}$

Pre-Calculus Name________________________________



# Addition and Subtraction Formulas Notes

Date________________ Period____

**Use the angle sum identity to find the exact value of each.**

1) $\sin 195°$

$\frac{\sqrt{2} - \sqrt{6}}{4}$

2) $\cos 75°$ $\frac{\sqrt{6} - \sqrt{2}}{4}$

**Use the angle difference identity to find the exact value of each.**

3) $\tan \frac{17\pi}{12}$

$2 + \sqrt{3}$

4) $\cos -\frac{13\pi}{12}$ $\frac{-\sqrt{2} - \sqrt{6}}{4}$

**Use an addition or subtraction formula to write the expression as a trigonometric function of one number, and then find its exact value.**

5) $\cos 10° \cos 80° - \sin 10° \sin 80°$

$0$

6) $\sin 23° \cos 37° + \sin 37° \cos 23°$ $\dfrac{\sqrt{3}}{2}$

7) $\cos \dfrac{19\pi}{12} \cos \dfrac{\pi}{4} + \sin \dfrac{19\pi}{12} \sin \dfrac{\pi}{4}$

$-\dfrac{1}{2}$

8) $\left( \tan \pi + \tan \dfrac{\pi}{6} \right) \div \left(1 - \tan \pi \tan \dfrac{\pi}{6} \right)$ $\dfrac{\sqrt{3}}{3}$

**Prove the identity.**

9) $\cos \left( \theta - \dfrac{\pi}{2} \right) = \sin \theta$

10) $\sin \left( \dfrac{\pi}{2} - \theta \right) = \cos \theta$

11) $\cot(x+y) = (\cot x \cot y - 1) \div (\cot x + \cot y)$

**Assuming** $\sin x = \frac{3}{5}$ **if** $\frac{\pi}{2} < x < \pi$ **and** $\cos y = -\frac{5}{13}$ **if** $\pi < y < \frac{3\pi}{2}$

12) $\sin(x + y)$

$\frac{33}{65}$

13) $\cos(x + y)$ $\frac{56}{65}$

14) $\tan(x - y)$ $\frac{63}{16}$

15) $\sec(x - y)$ $-\frac{65}{16}$